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The Significance and Interpretation of Recovery Data

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Last update: March 8, 2026

Significance of recovery data

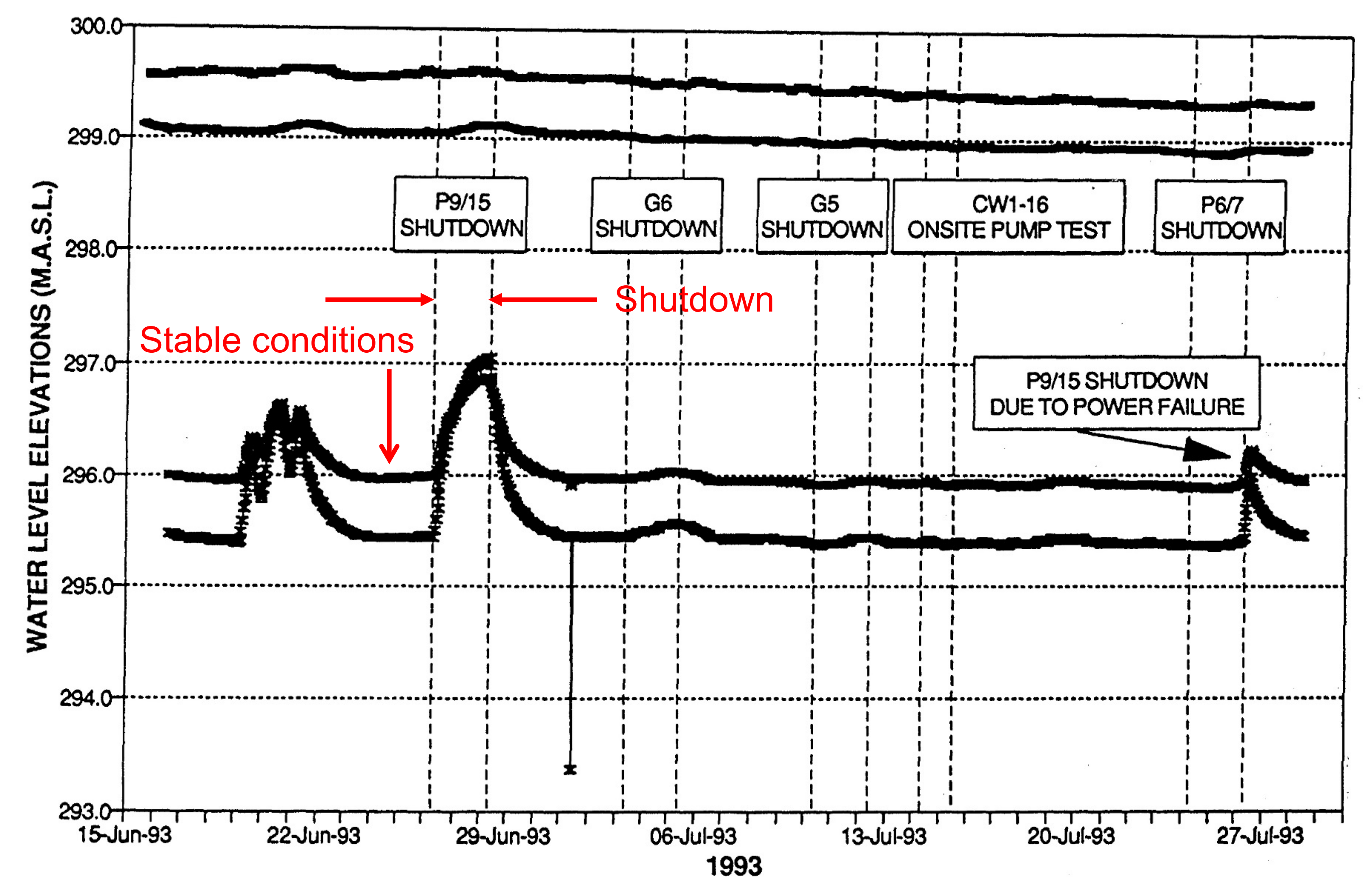
1. Sometimes all we get are recovery data.
2. Recovery data are “smoother”.
3. Recovery data can be used to assess whether there was a background trend in water levels during a pumping test.
4. Recovery data may provide a useful check on the interpretations of drawdowns.
5. Recovery data can provide insights that the drawdown data do not.
6. Recovery data can be used to extend the effective duration of pumping.

Fact:

Sometimes it is only feasible to collect recovery data. This is particularly true for the case of operating municipal wellfields.

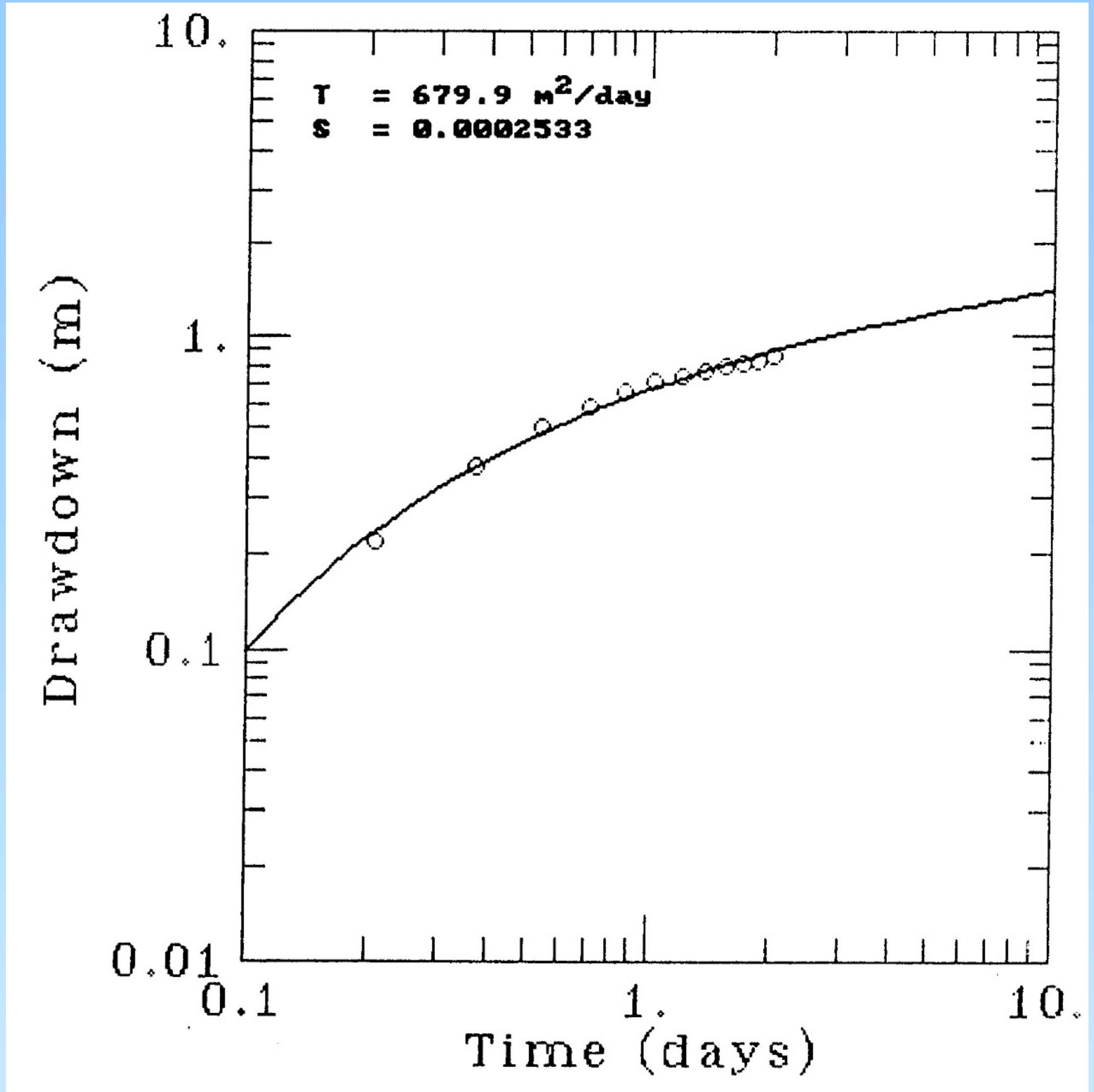
With a bit of planning and communication with municipal operators, it may be possible to turn a scheduled shutdown into an effective test.

City of Cambridge, Ontario: Well shutdown test



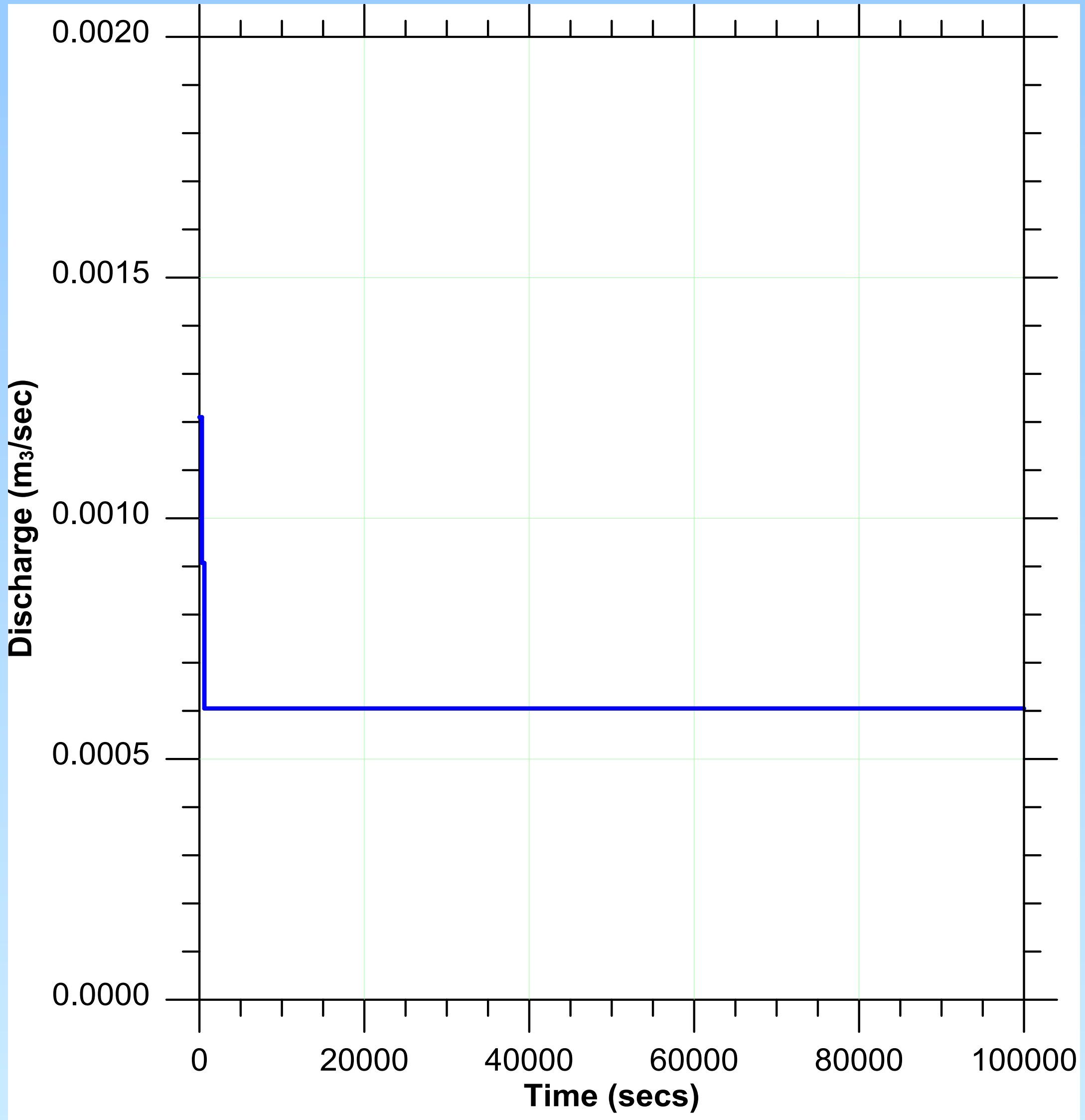
Since conditions were stable before the shutdown, in this case the shutdown test was in fact a pumping test run in reverse.

Interpretation of well shutdown test

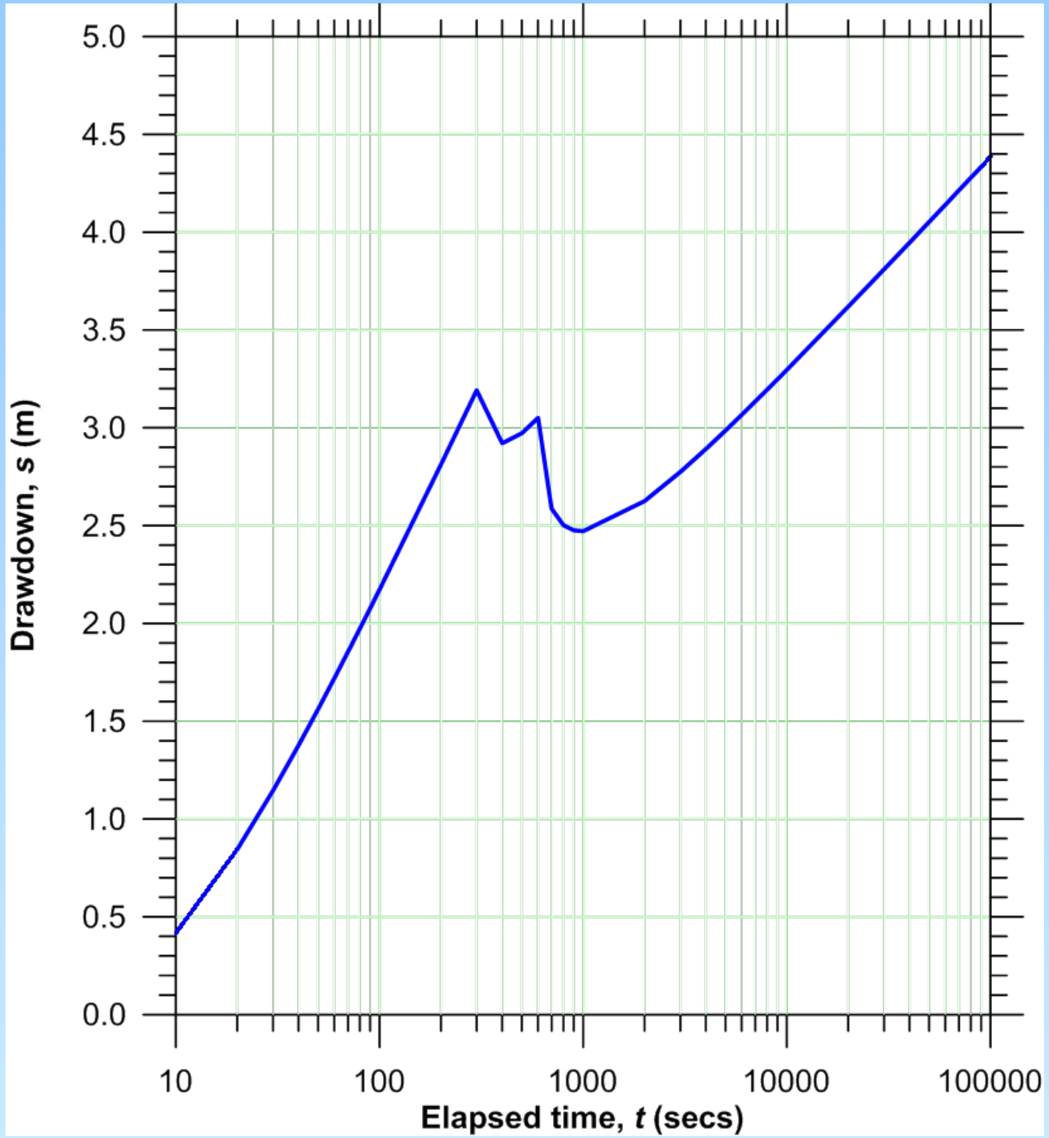


Fact: Recovery data are generally smoother than drawdown data

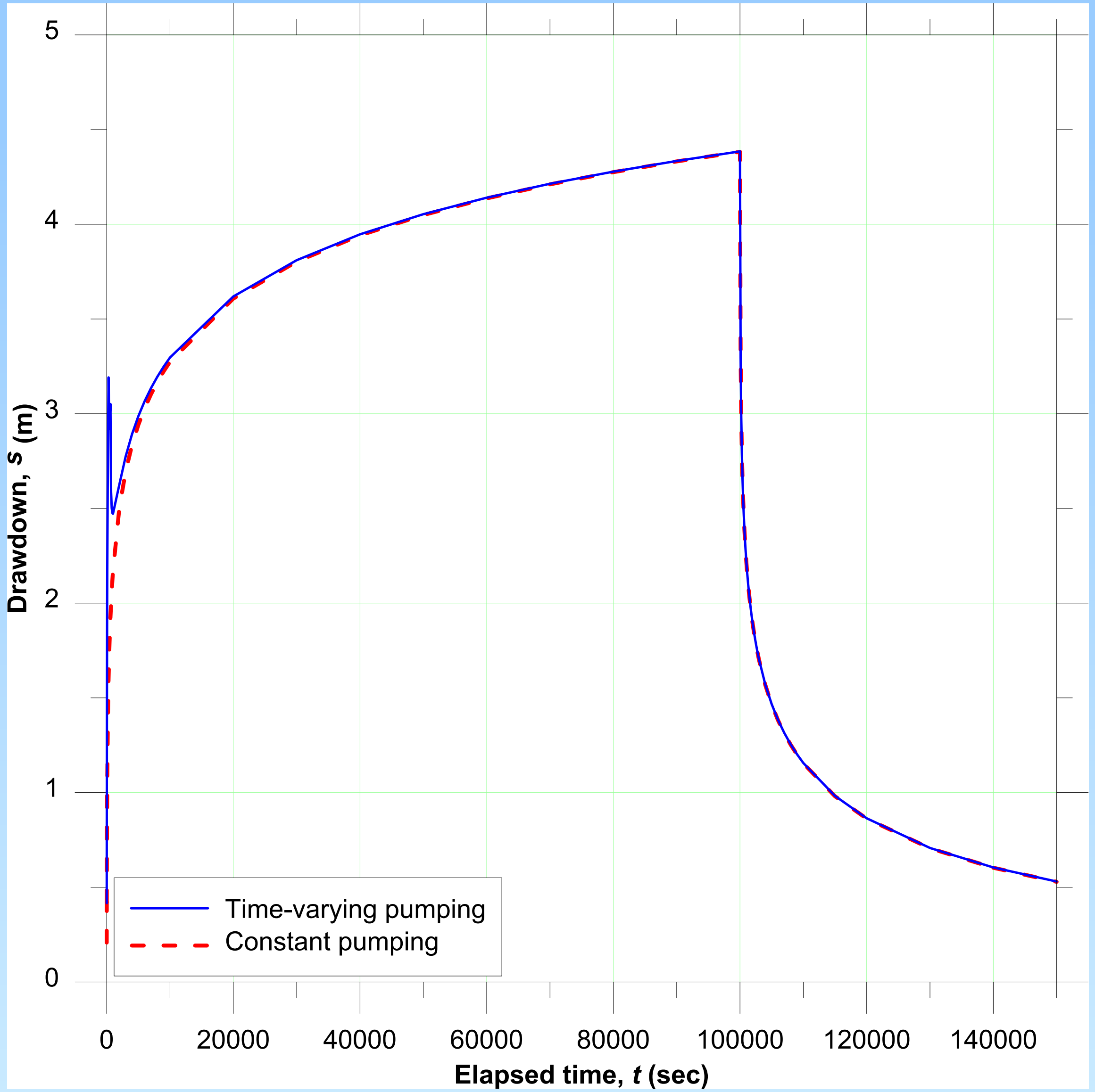
Smoothing effect: Example #1



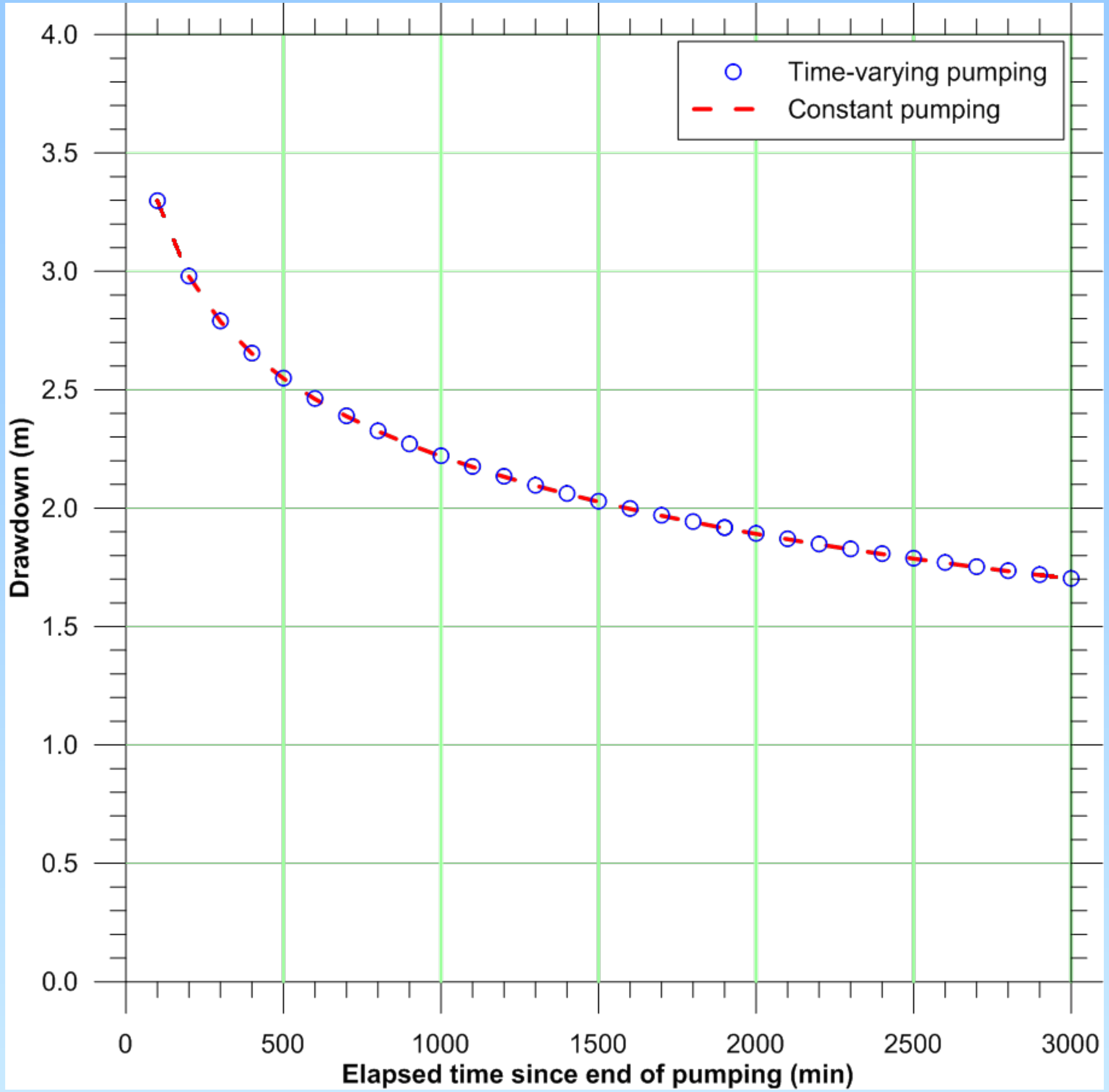
Drawdown record



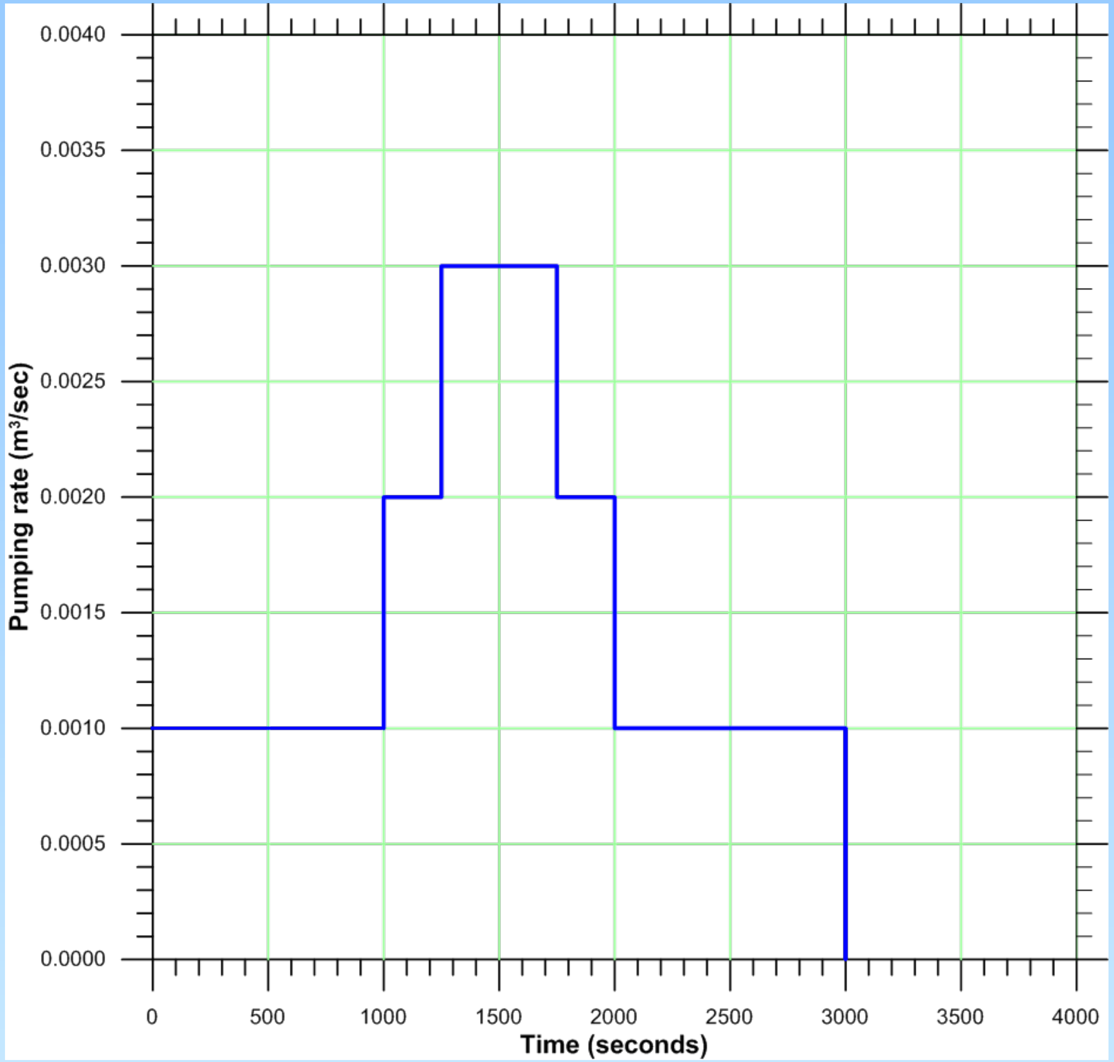
Complete time-drawdown record



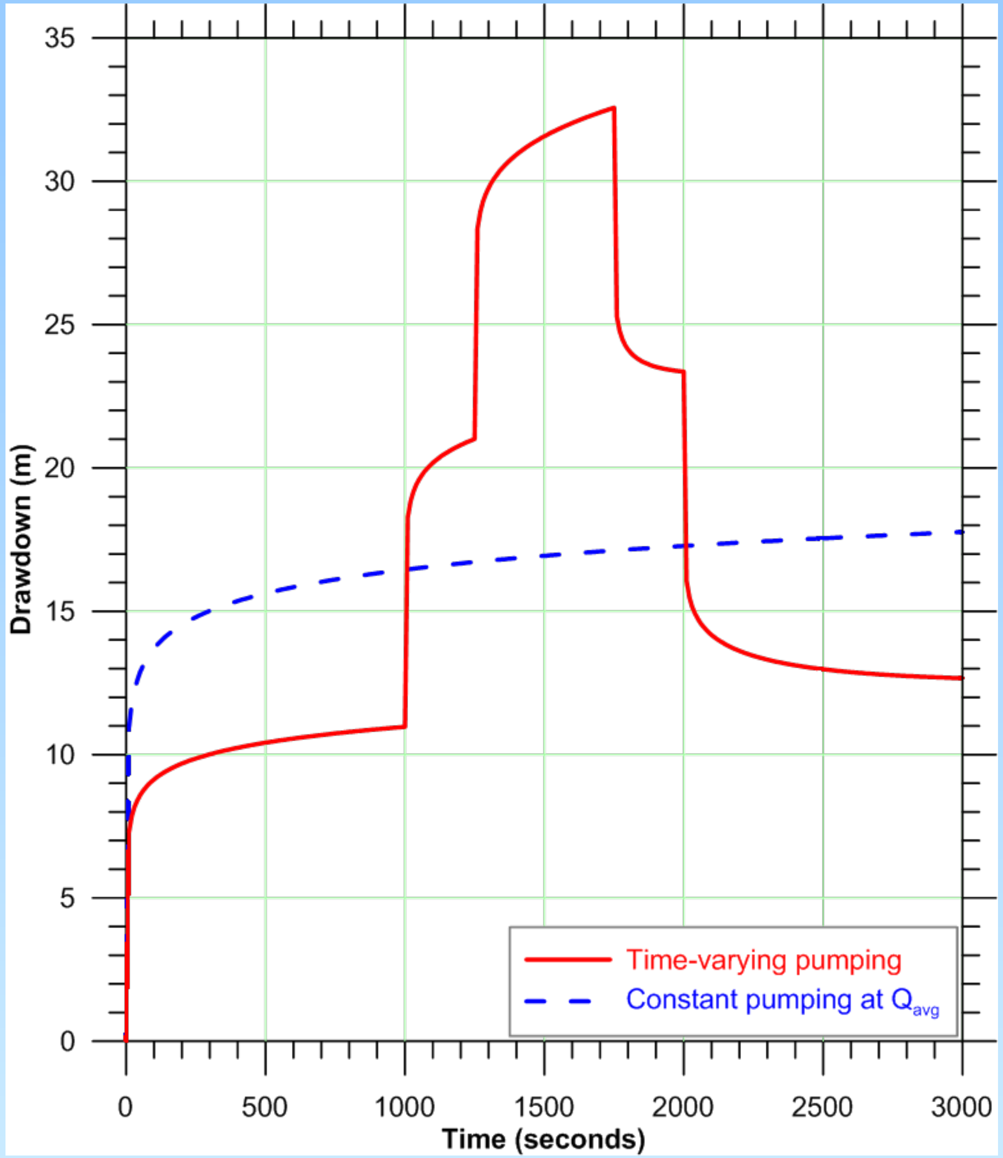
Plot of recovery



Smoothing effect: Example #2

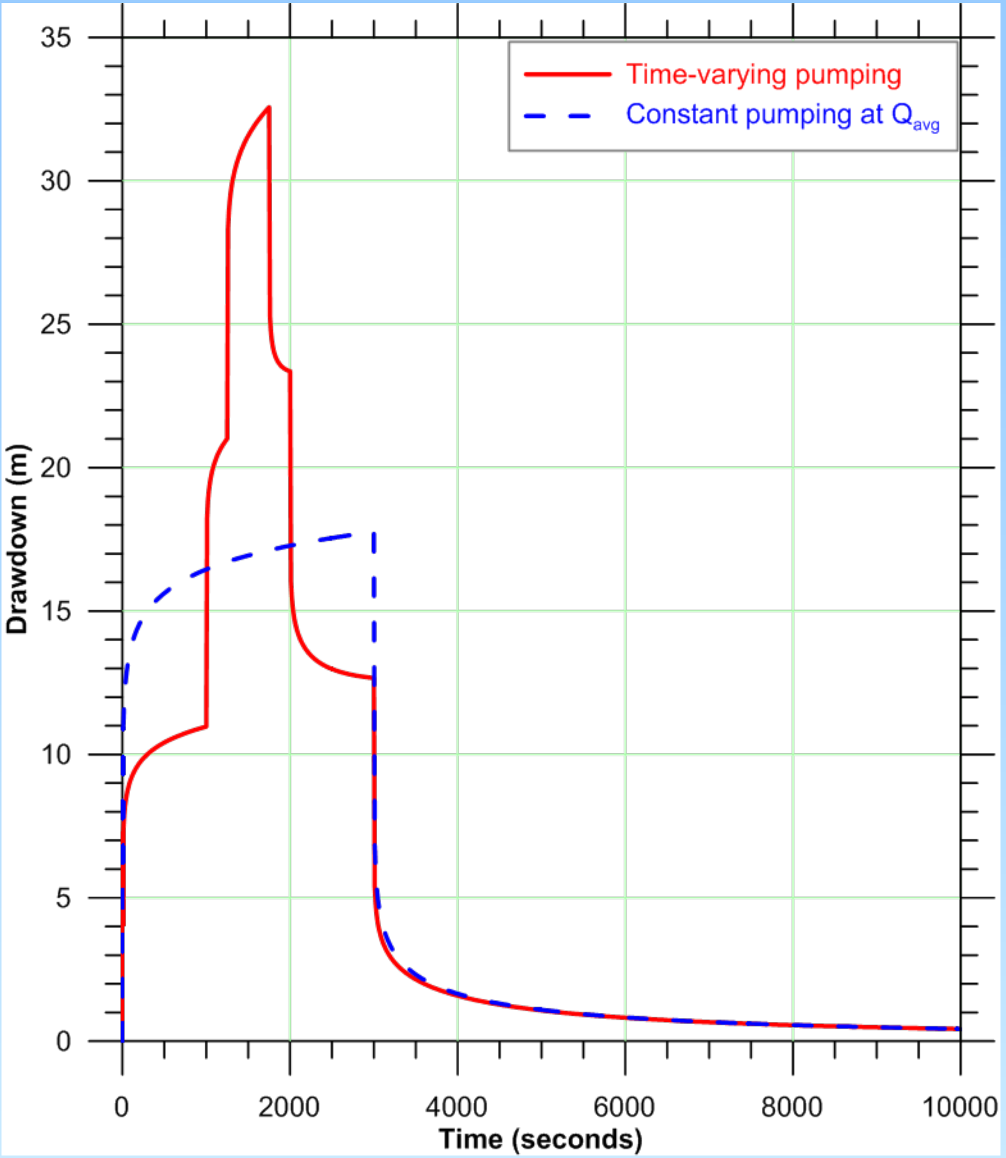


Drawdown record

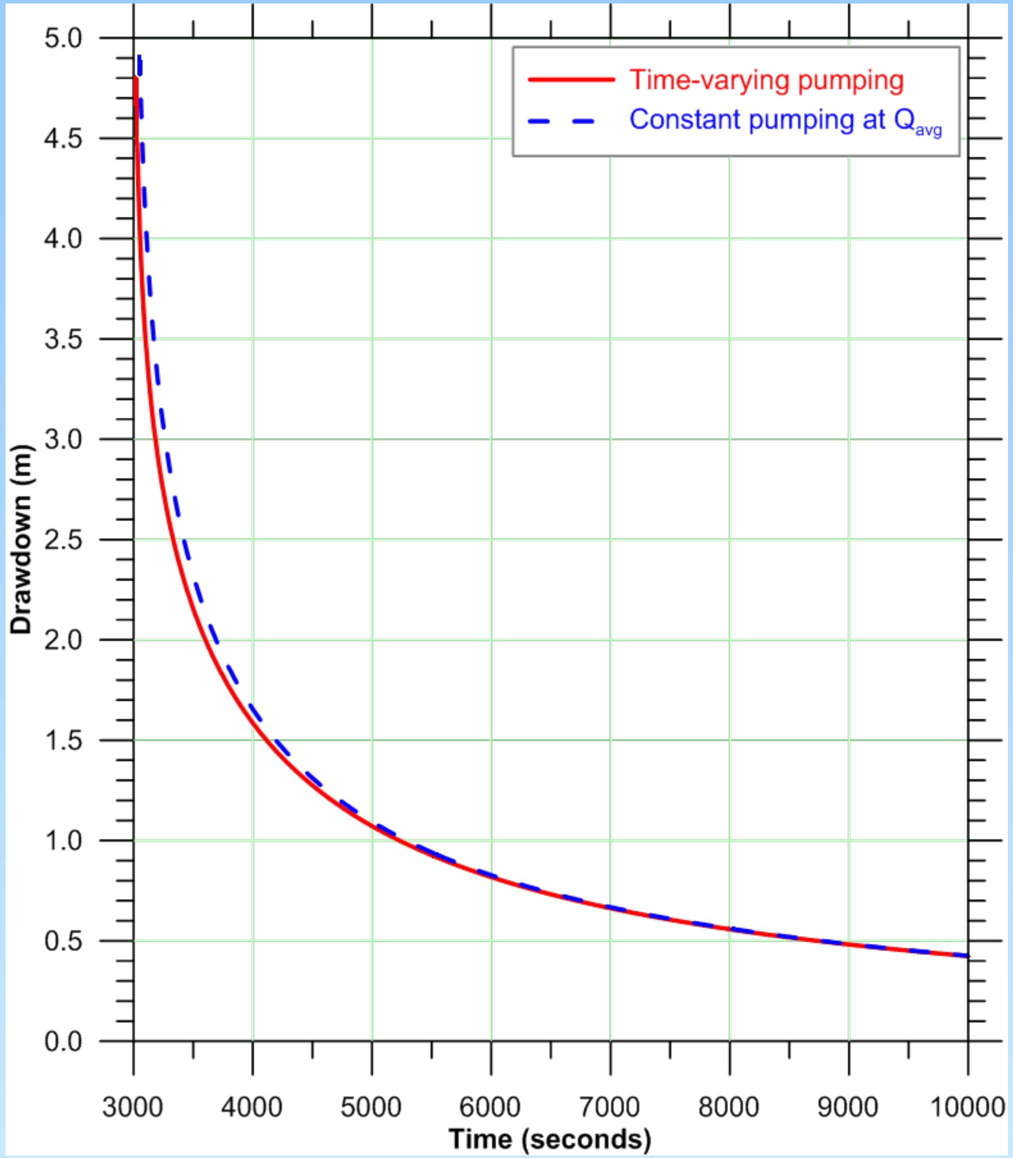


Q_{avg} is defined as that constant rate for which the same volume of water is extracted by the end of pumping.

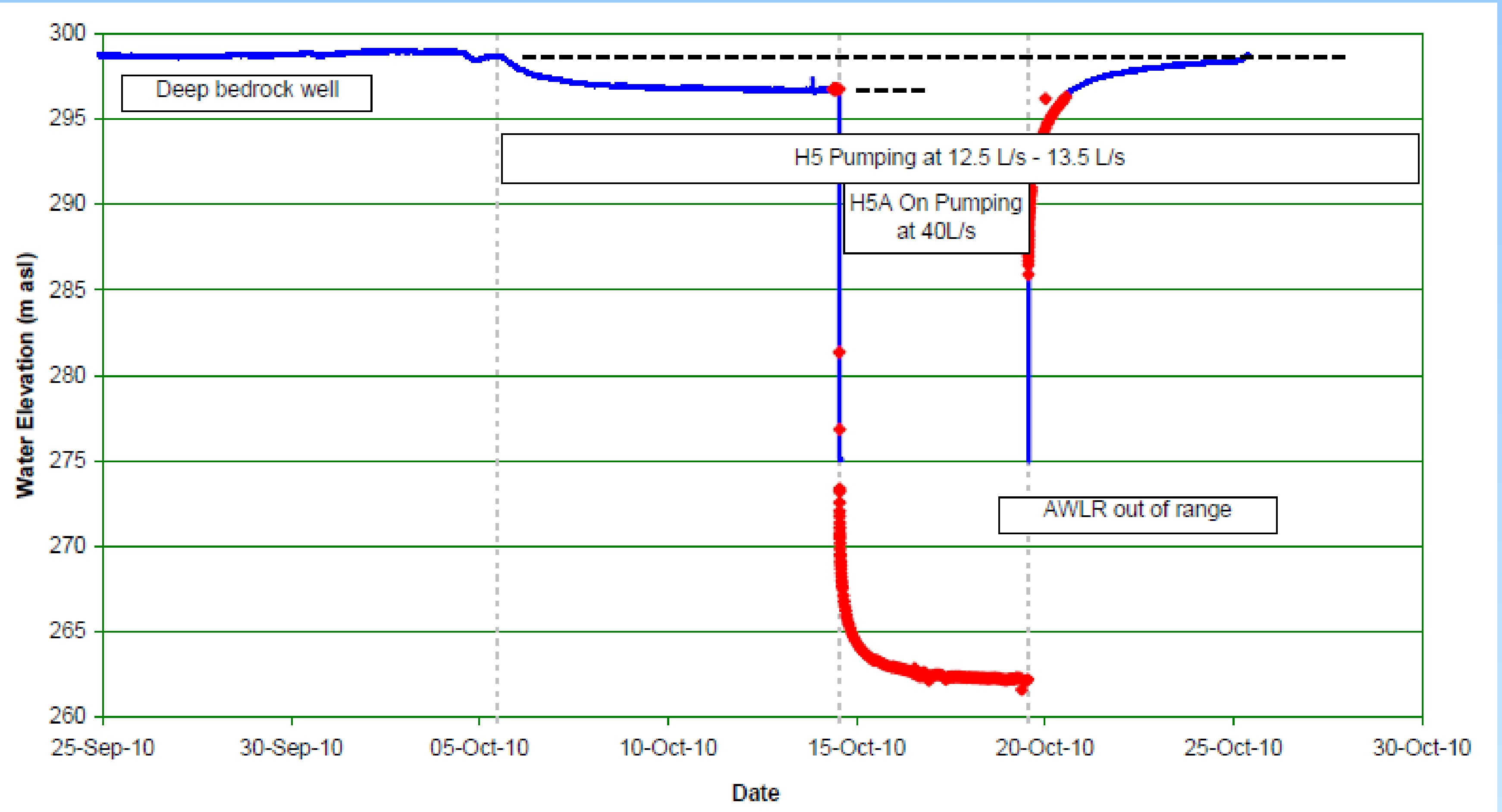
Complete time-drawdown record



Plot or recovery

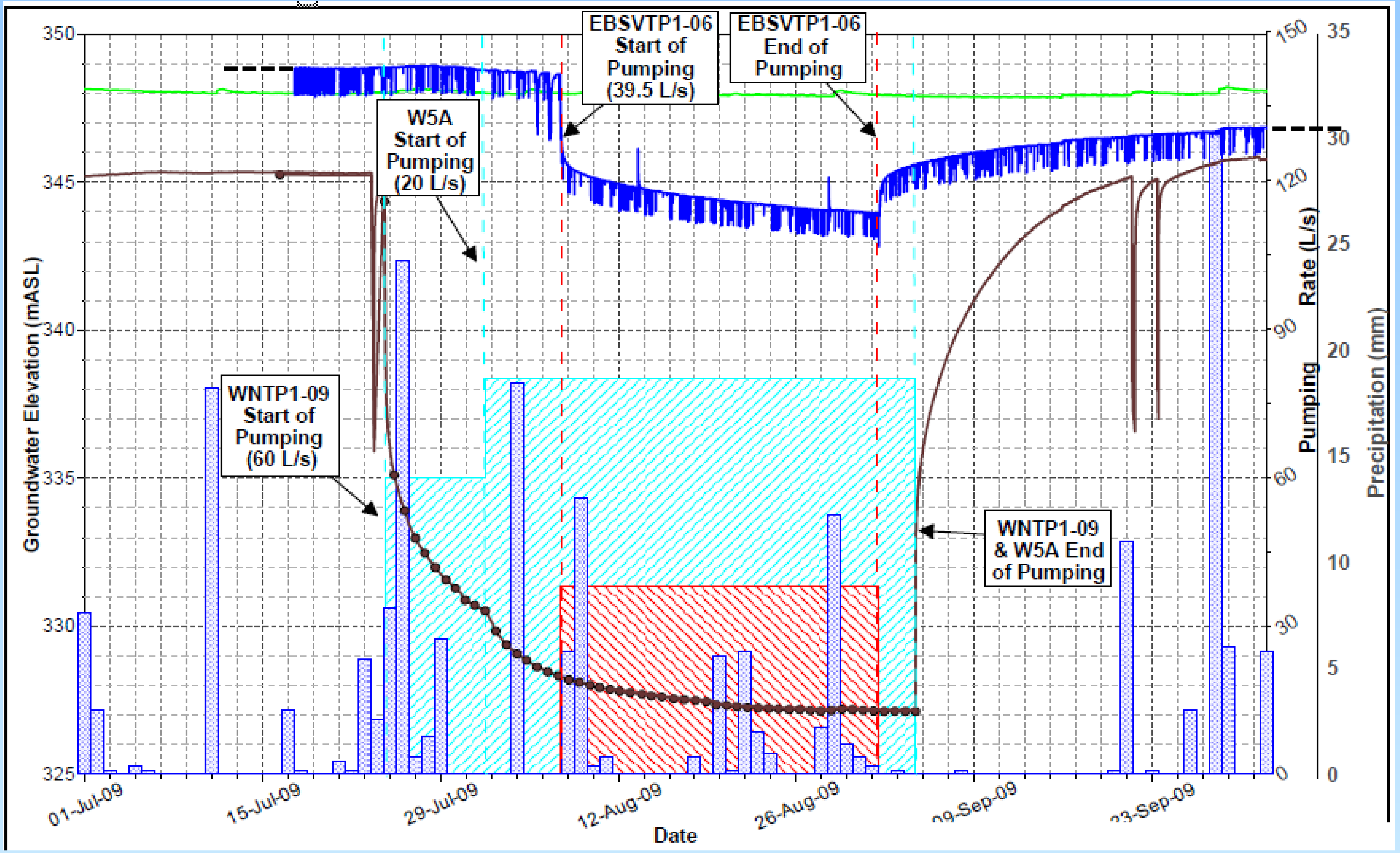


Use of recovery data to check on background trends during pumping



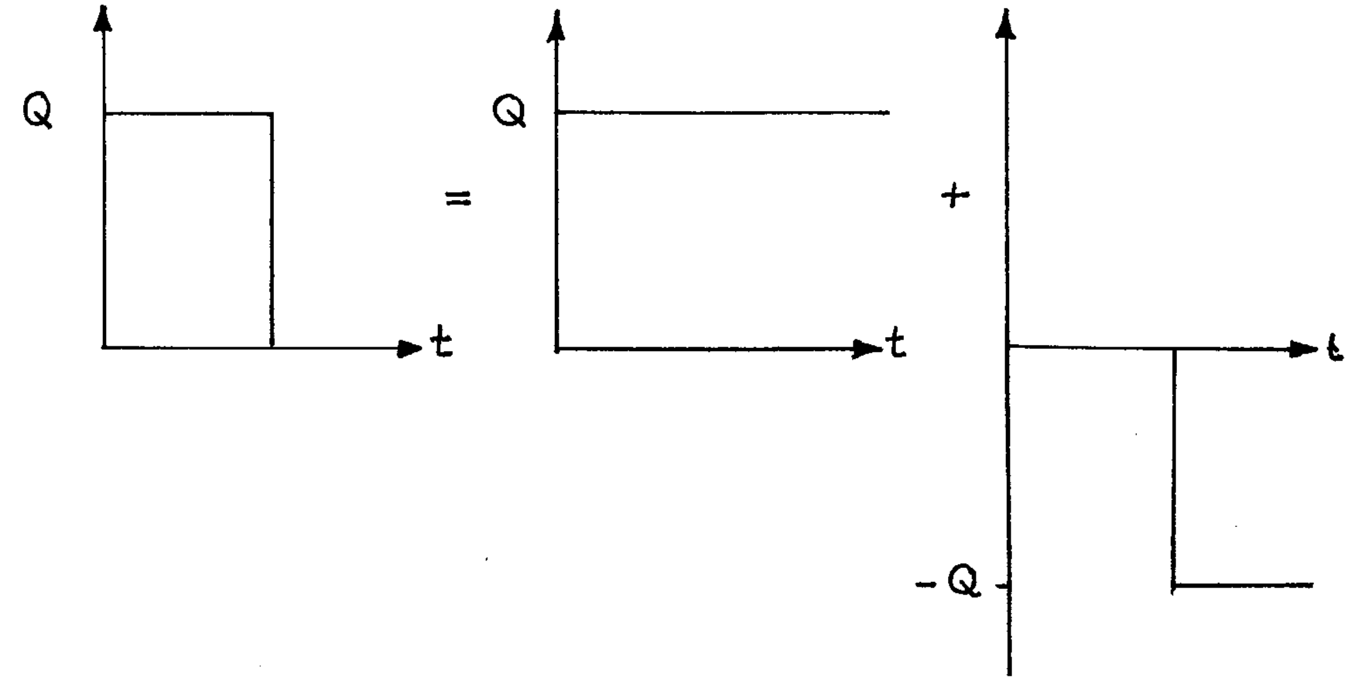
Pumping from two wells. What are the drawdowns caused by H5A?

Use of recovery data to check on background trends during pumping (2)



Pumping from three wells. What are the drawdowns caused by WNTP1-09?

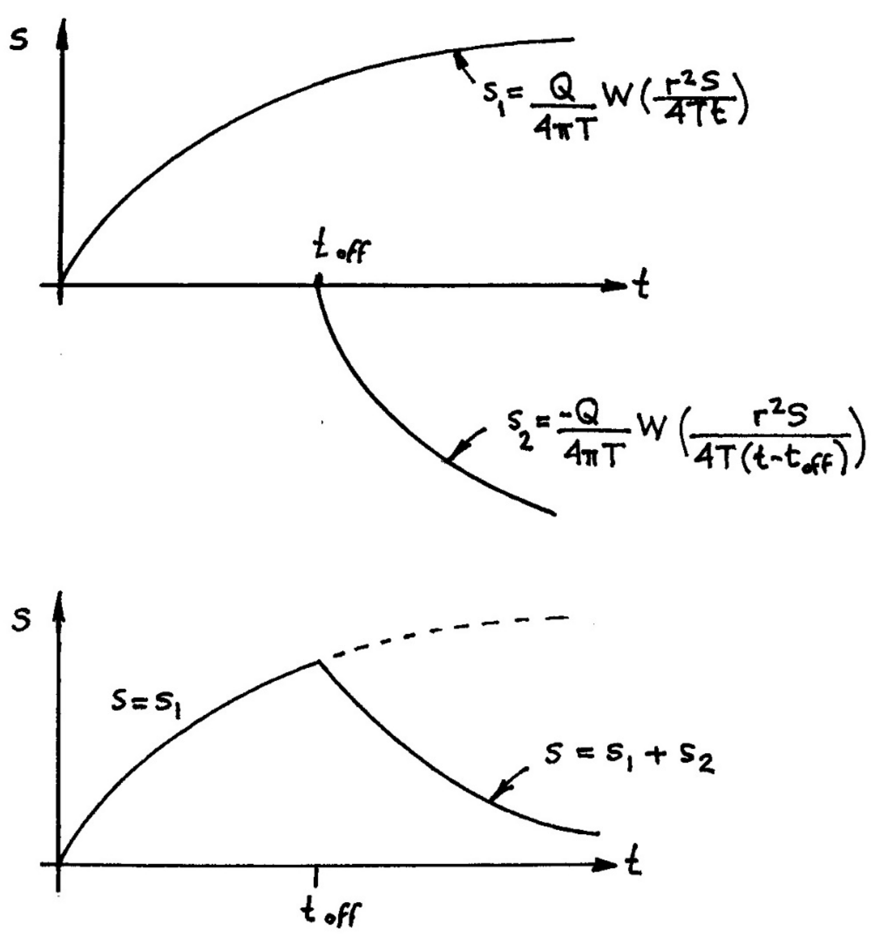
Interpretation of recovery data: Principle of Superposition



Superposition is valid for any linear aquifer model.

Superposition: Theis model

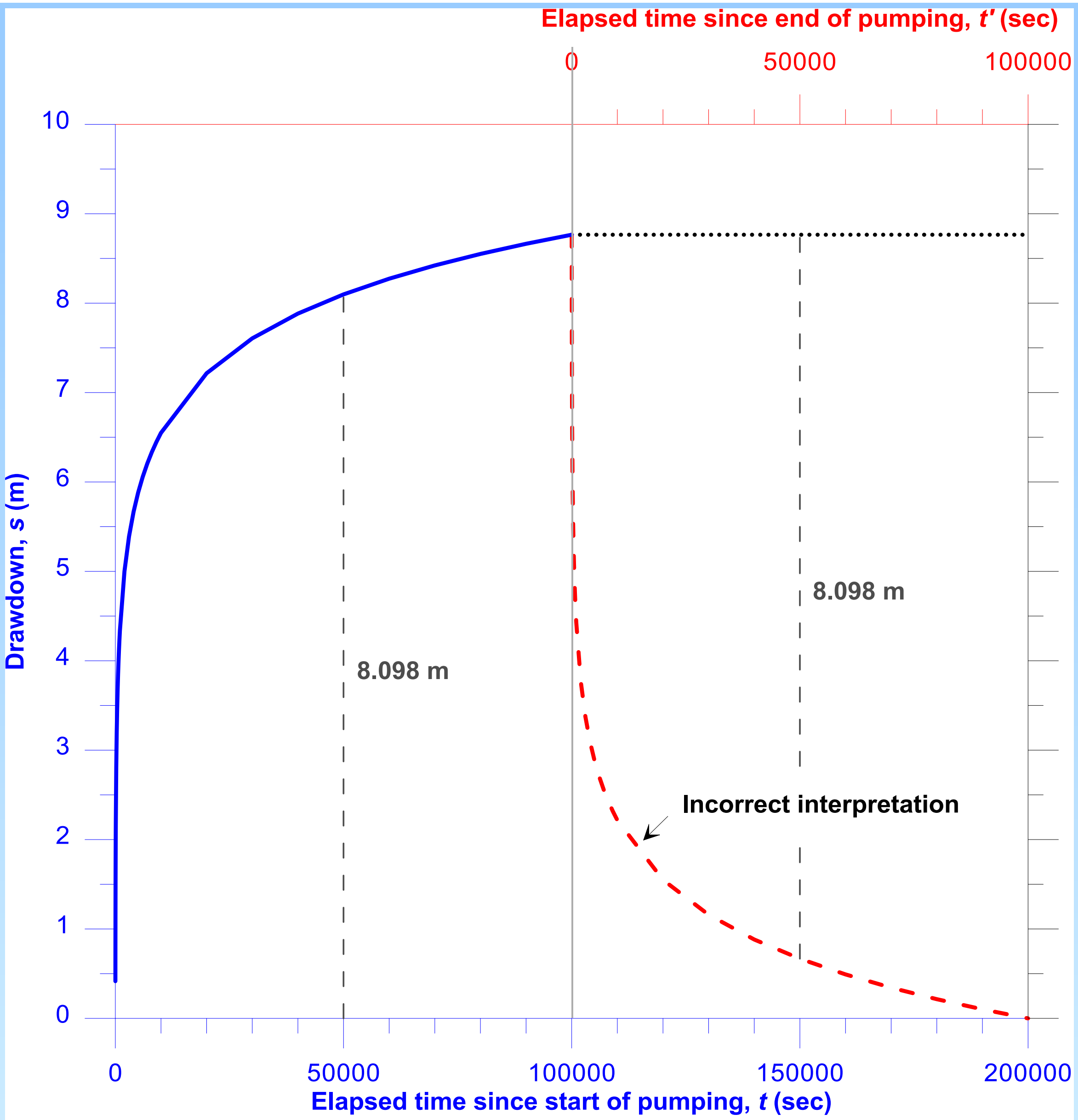
$$s(r,t) = \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4Tt}\right) - \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4T(t - t_{off})}\right)$$



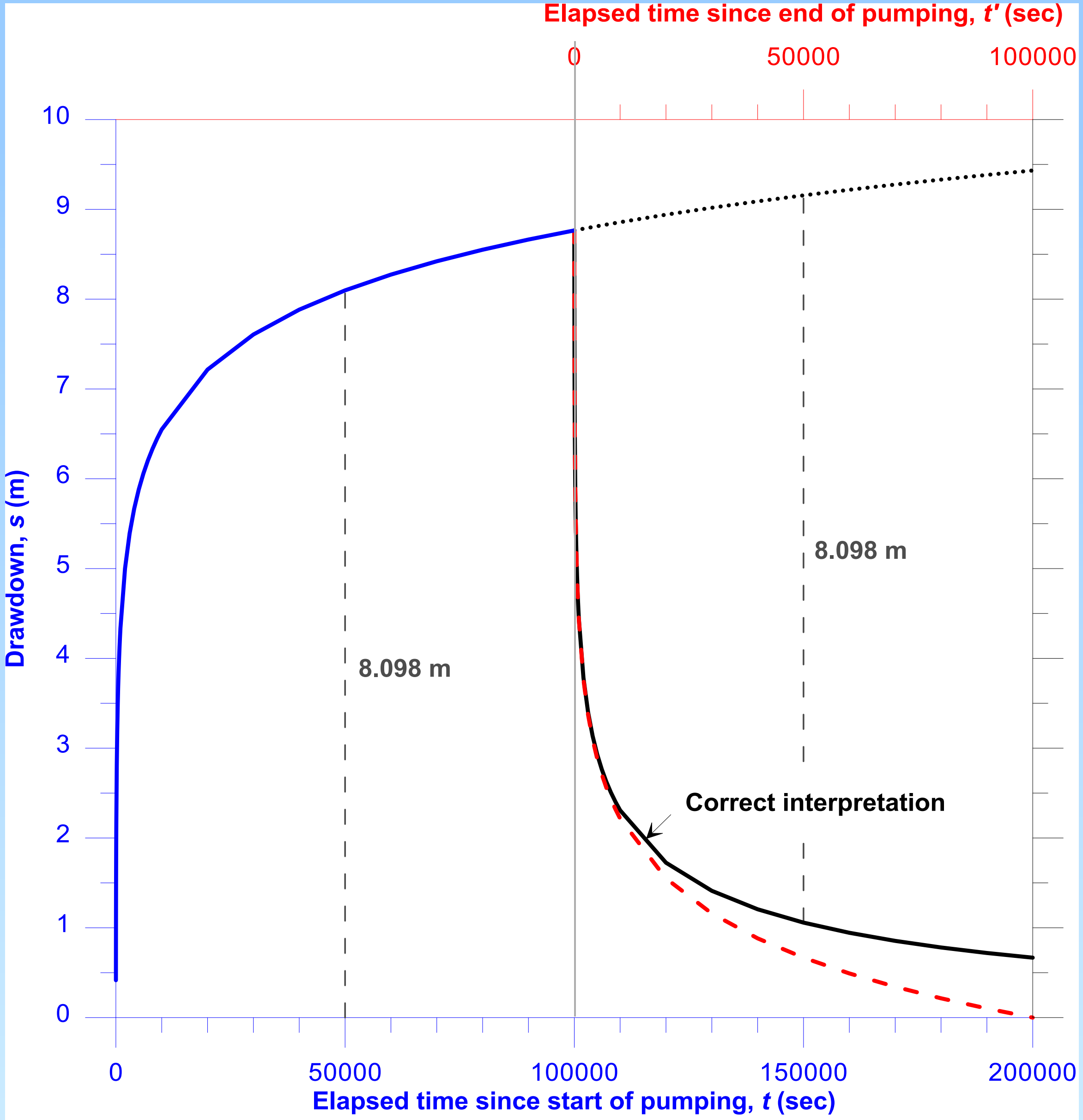
A common misconception:

Transient recovery is usually not a mirror image of drawdown

Incorrect



Correct



Recovery = Drawdown only if water levels have stabilized by the end of pumping.

Cooper-Jacob (1945) approximation

Recovery following pumping at a constant rate

$$s(r,t) = \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4Tt}\right) - \frac{Q}{4\pi T} W\left(\frac{r^2 S}{4T(t - t_{off})}\right)$$

Replacing *W*(*u*) with its approximation:

$$s(r,t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln\left\{\frac{r^2 S}{4Tt}\right\}\right] - \frac{Q}{4\pi T}\left[-0.5772 - \ln\left\{\frac{r^2 S}{4T(t - t_{off})}\right\}\right]$$

$$s(r,t) = \frac{Q}{4\pi T}\left[-\ln\left\{\frac{r^2 S}{4Tt}\right\} + \ln\left\{\frac{r^2 S}{4T(t - t_{off})}\right\}\right]$$

A little bit more algebra:

$$s(r,t) = \frac{Q}{4\pi T}\ln\left\{\frac{\left(\frac{r^2 S}{4T(t - t_{off})}\right)}{\left(\frac{r^2 S}{4Tt}\right)}\right\}$$

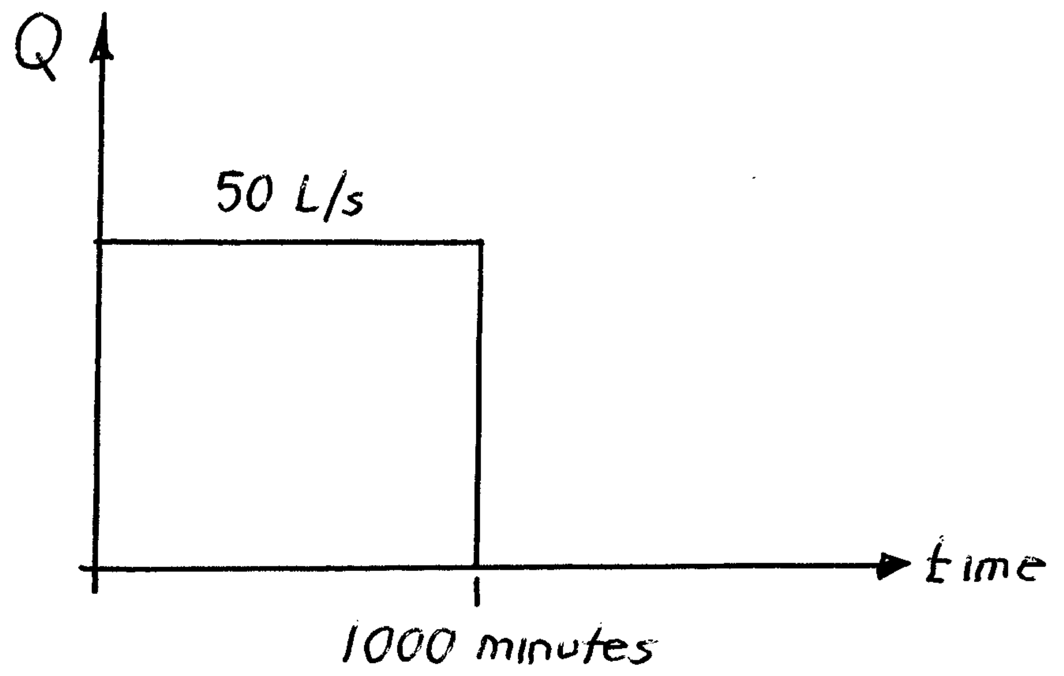
$$= 2.303\frac{Q}{4\pi T}\log_{10}\left\{\frac{t}{t - t_{off}}\right\}$$

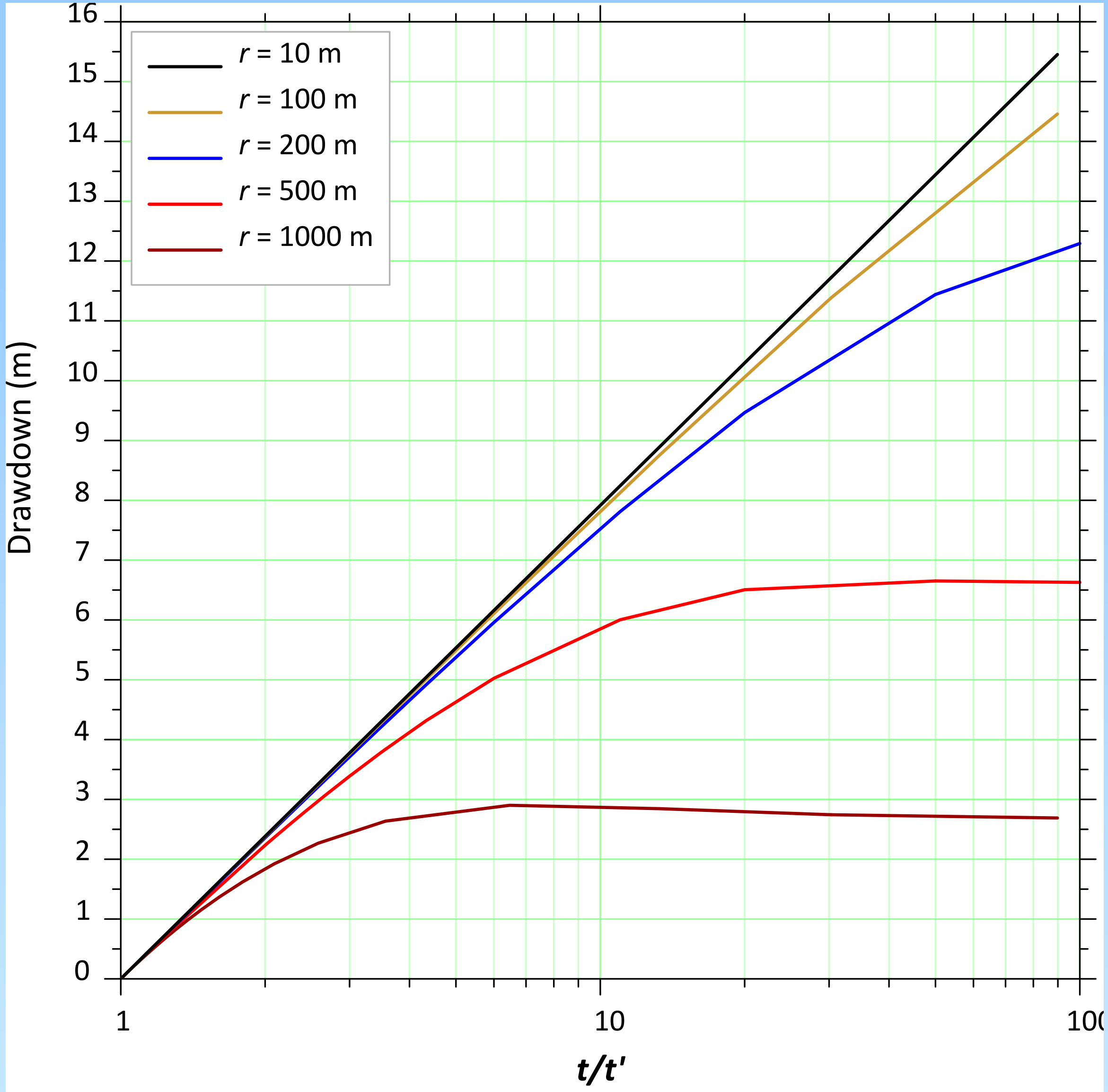
$$= 2.303\frac{Q}{4\pi T}\log_{10}\left\{\frac{t}{t'}\right\}$$

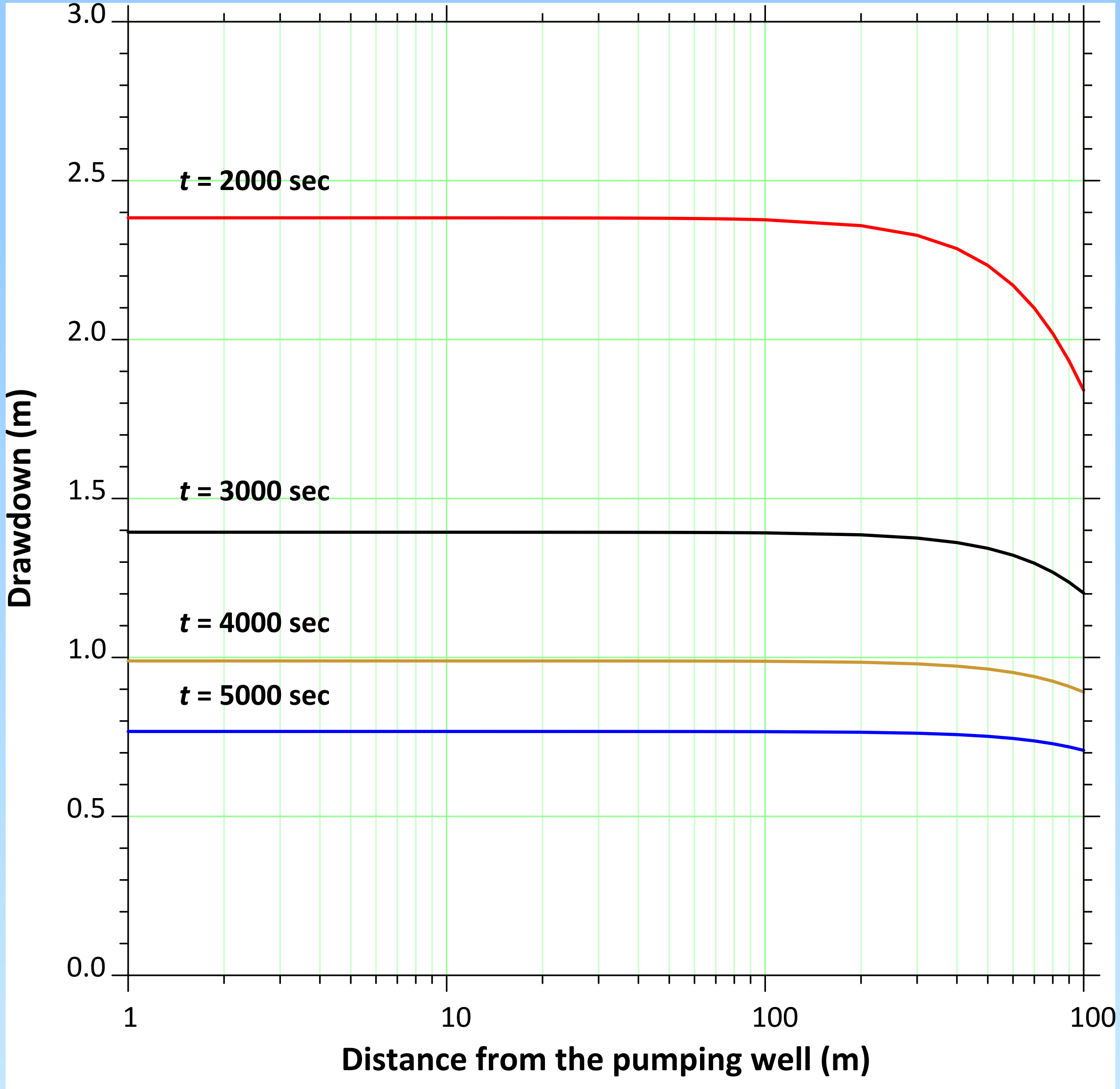
Where are *r* and *S*?

Example analysis

T = 100 m^2/d
S = 1.0×10^{-4}

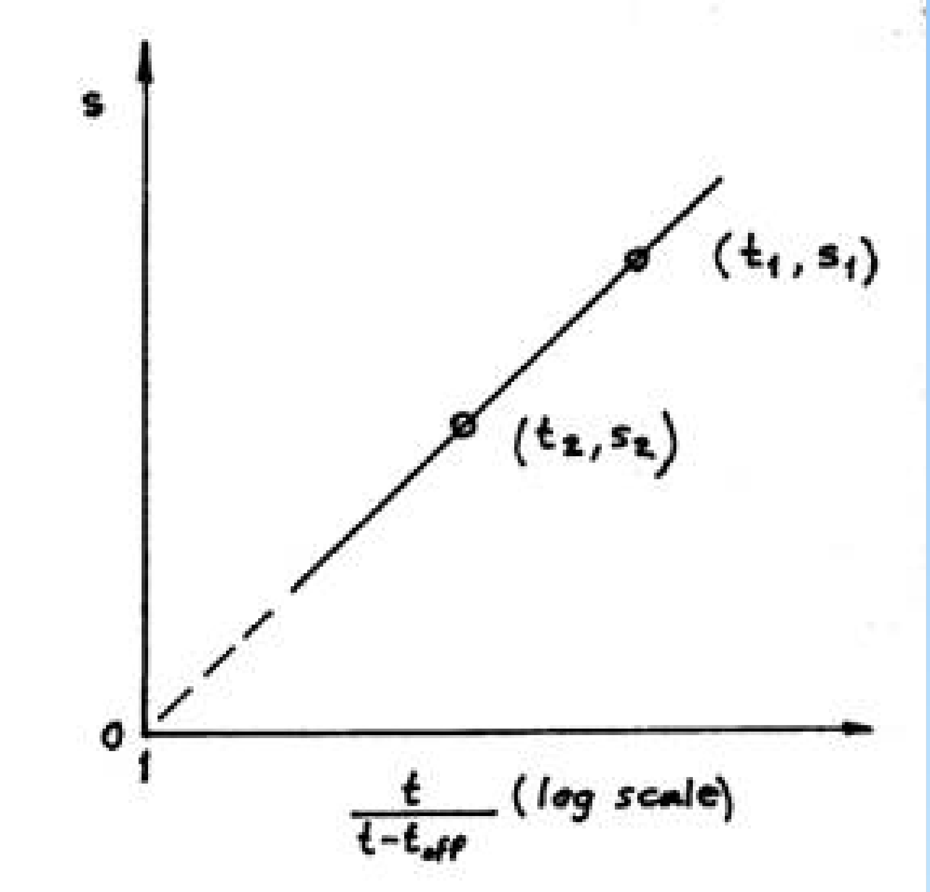



r = 10 m
r = 100 m
r = 200 m
r = 500 m
r = 1000 m
Drawdown (m)
t/t'
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
1
10
100



3.0
2.5
2.0
1.5
1.0
0.5
0.0
Drawdown (m)
t = 2000 sec
t = 3000 sec
t = 4000 sec
t = 5000 sec
1
10
100
100
Distance from the pumping well (m)

Cooper-Jacob straight-line recovery analysis



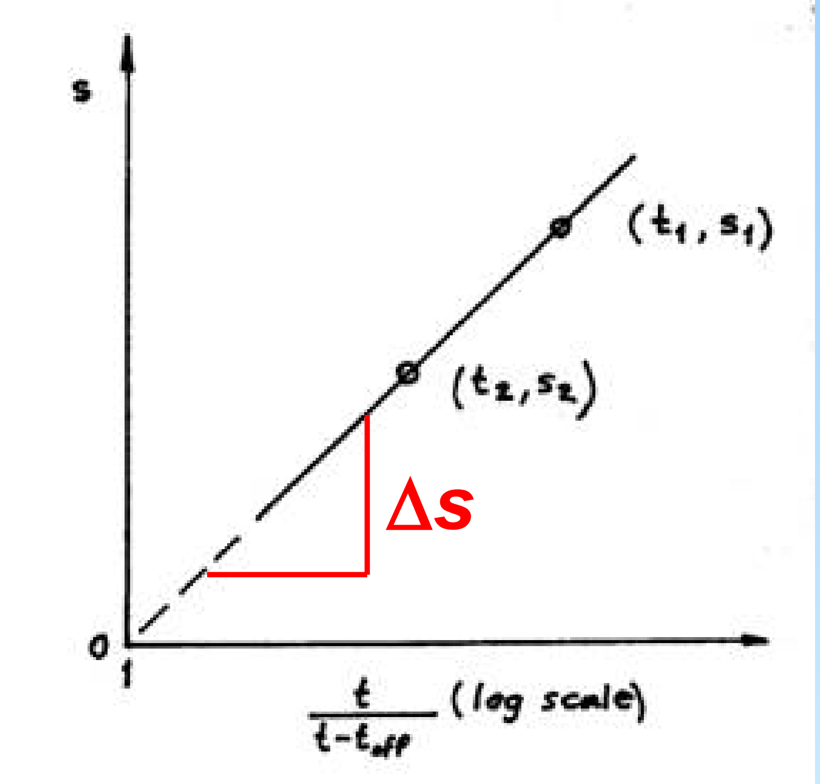
$$s(r,t_1) = 2.303\frac{Q}{4\pi T}\log_{10}\left\{\frac{t_1}{t_1 - t_{off}}\right\}$$

$$s(r,t_2) = 2.303\frac{Q}{4\pi T}\log_{10}\left\{\frac{t_2}{t_2 - t_{off}}\right\}$$

$$s(r,t_1) - s(r,t_2) = 2.303\frac{Q}{4\pi T}\log_{10}\left\{\frac{\left\{\frac{t_1}{t_1 - t_{off}}\right\}}{\left\{\frac{t_2}{t_2 - t_{off}}\right\}}\right\}$$

Straight-line analysis (2)

$$T = 2.303\frac{Q}{4\pi}\frac{\log_{10}\left\{\frac{\left(\frac{t}{t-t_{off}}\right)_1}{\left(\frac{t}{t-t_{off}}\right)_2}\right\}}{(s_1 - s_2)} = 2.303\frac{Q}{4\pi}\frac{1}{\Delta s}$$

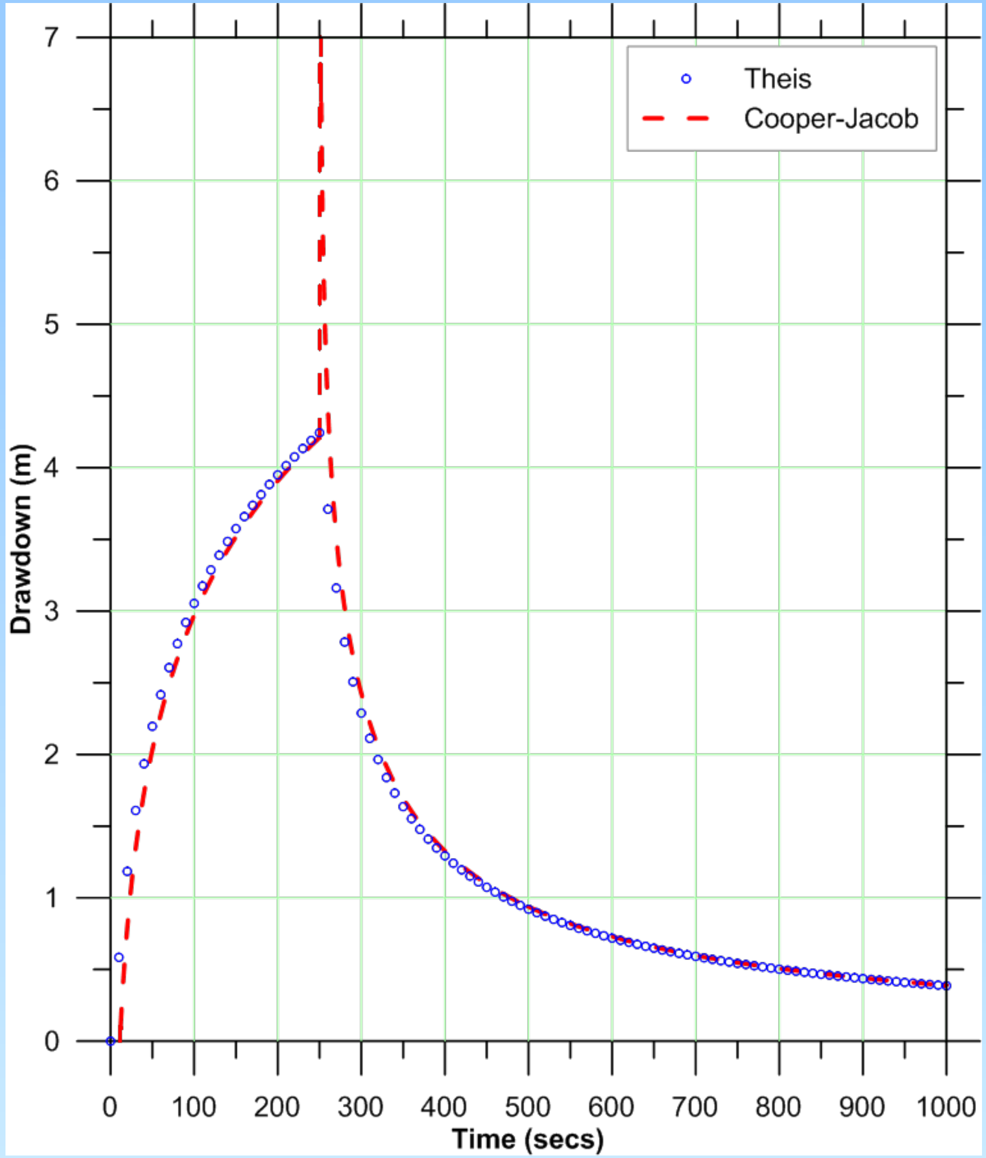


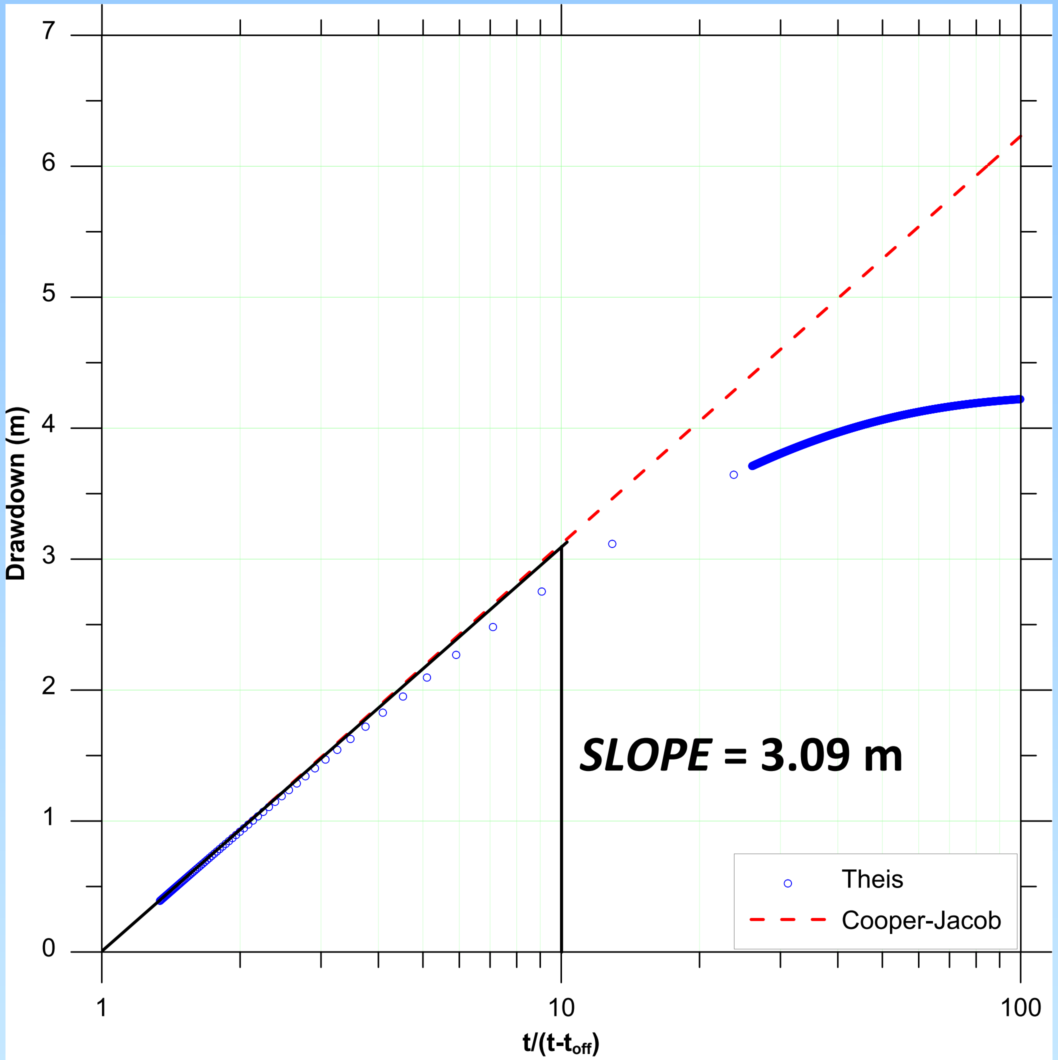
Denoting $t\text{-}t_{off} = t'$

$$\Delta s = \frac{\Delta\ \text{drawdown}}{\text{log cycle}\left\{\frac{t}{t'}\right\}}$$

Example

$T = 1.0 \times 10^{-4}$ m^2/s
$S = 1.0 \times 10^{-4}$
$Q = 1.7 \times 10^{-3}$ m^3/s





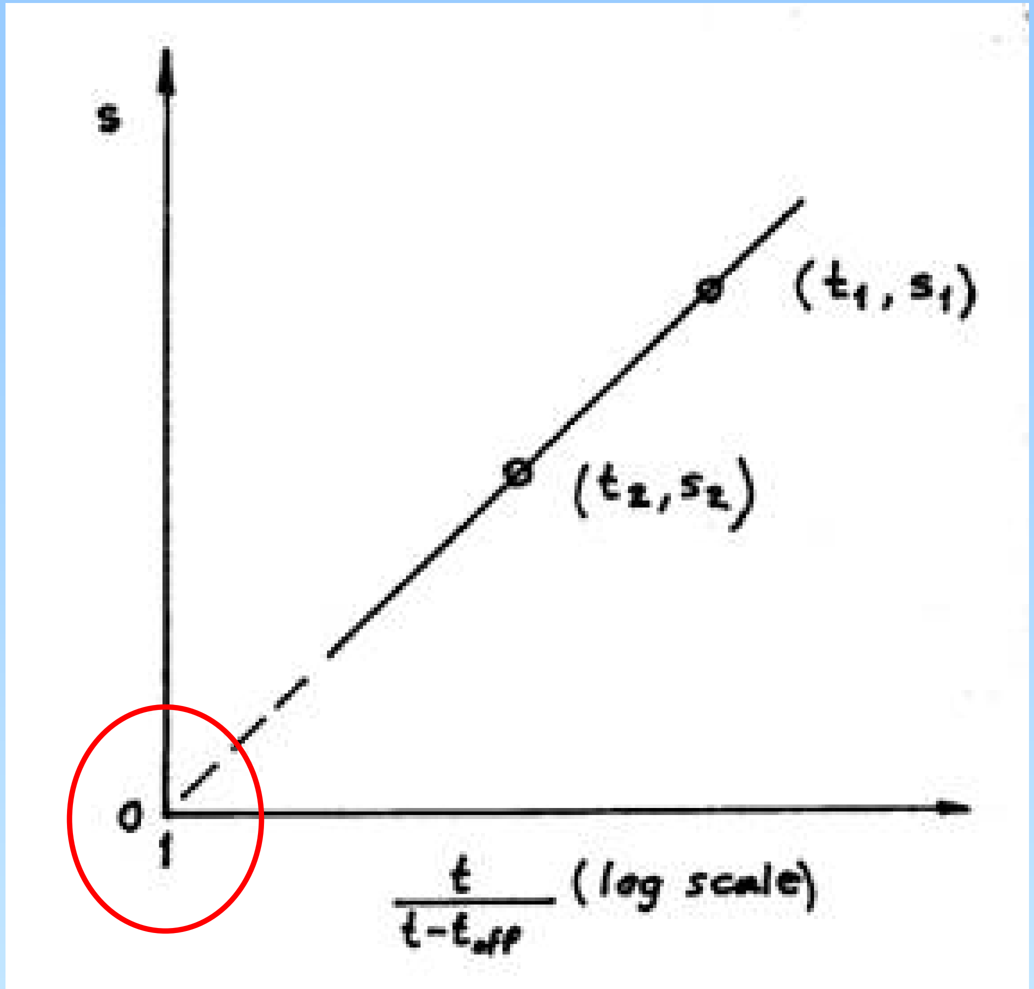
Estimation of transmissivity

$$T = 2.303 \frac{Q}{4\pi} \frac{1}{\Delta s}$$

$$T = 2.303 \frac{(1.7 \times 10^{-3} \text{ m}^3/\text{s})}{4\pi} \frac{1}{(3.09 \text{ m})}$$

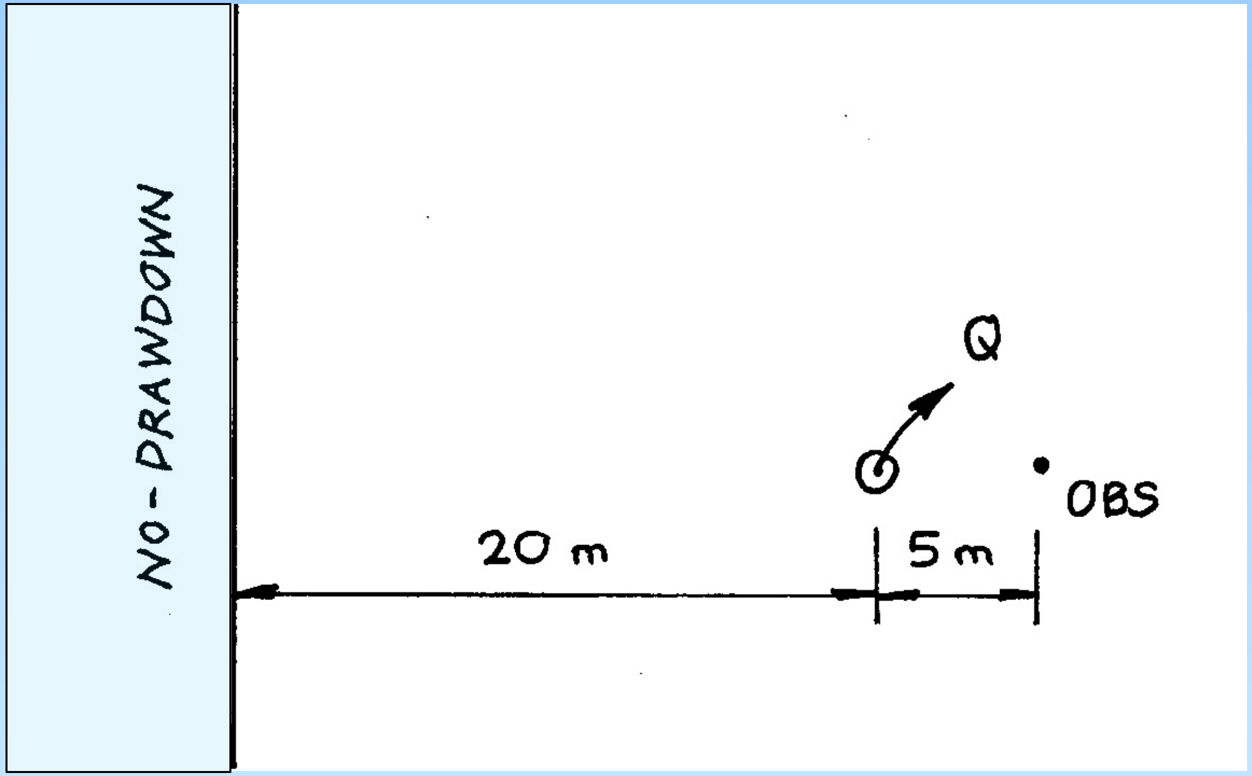
$$= \mathbf{1.0 \times 10^{-4} \ m^2/s}$$

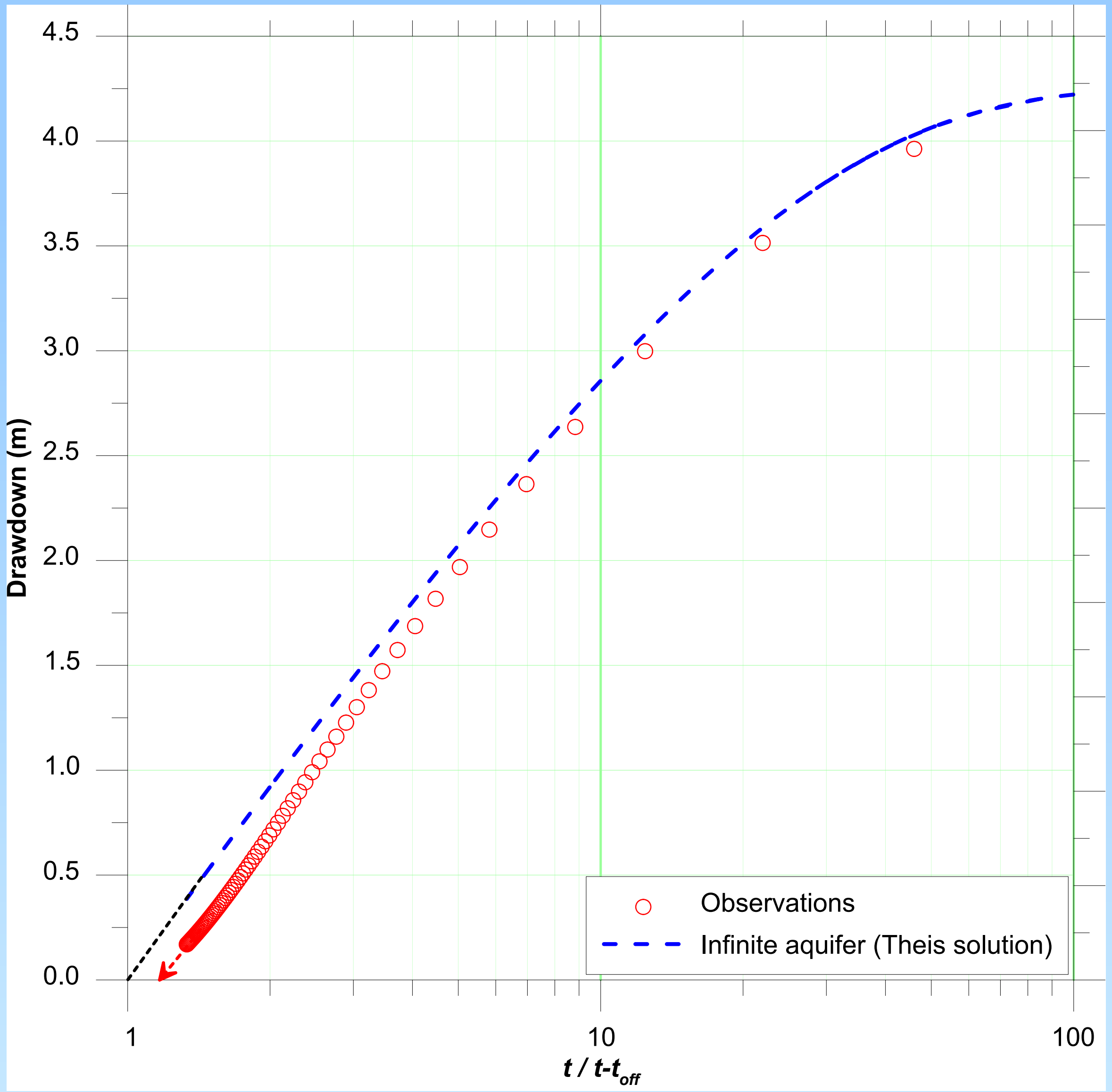
Asymptote for the Cooper-Jacob plot



$s(r,t) \rightarrow 0$ as $t \rightarrow \infty$

Example:
Pumping near a linear recharge boundary





Drawdown (m)
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1
10
100
t / t-t_off
Observations
Infinite aquifer (Theis solution)

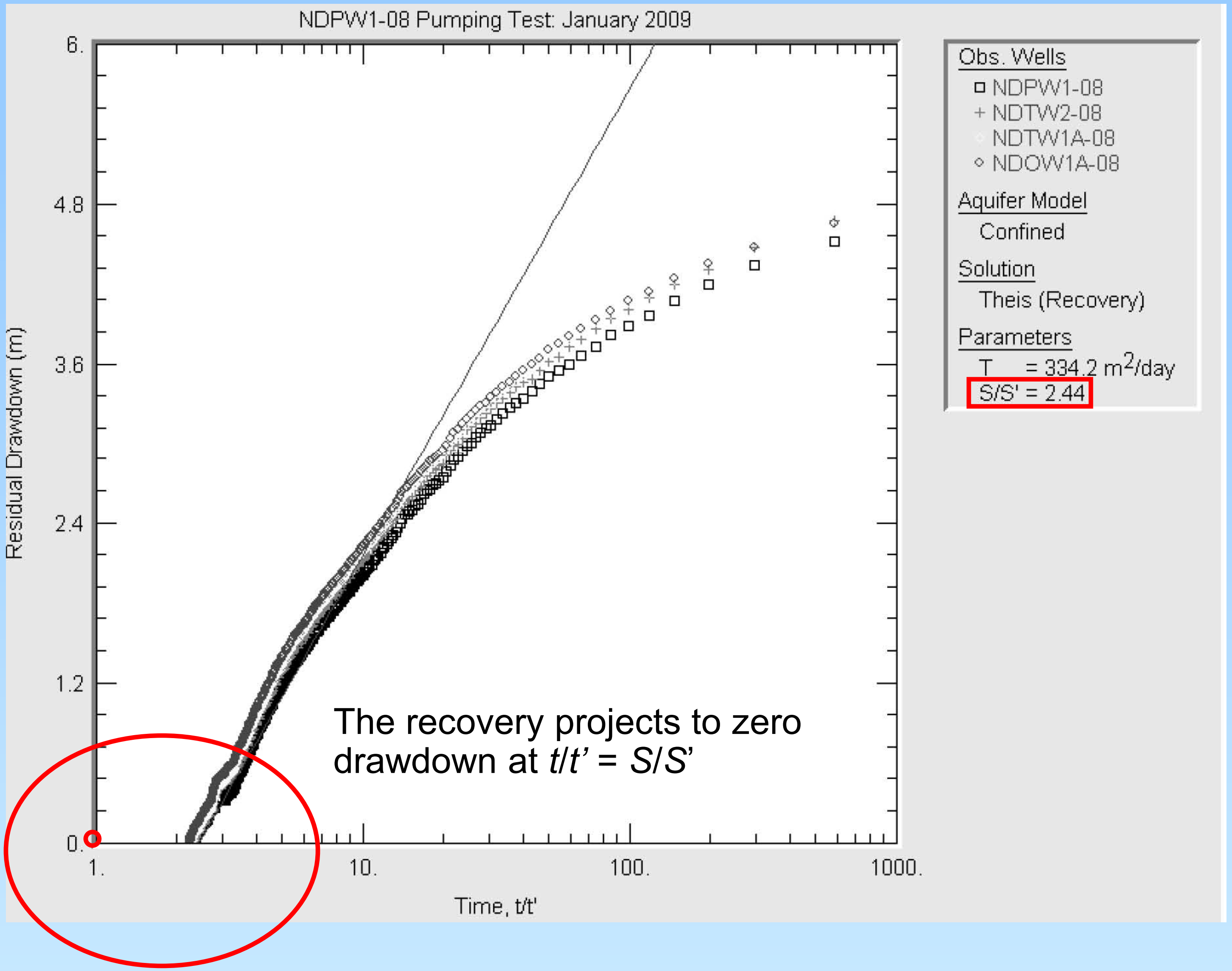
Something wrong but nevertheless useful

$$s(r,t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln\left\{\frac{r^2 S}{4Tt}\right\}\right] - \frac{Q}{4\pi T}\left[-0.5772 - \ln\left\{\frac{r^2 S'}{4T(t - t_{off})}\right\}\right]$$

$$s(r,t) = \frac{Q}{4\pi T}\ln\left\{\frac{\left(\frac{r^2 S'}{4T(t - t_{off})}\right)}{\left(\frac{r^2 S}{4Tt}\right)}\right\}$$

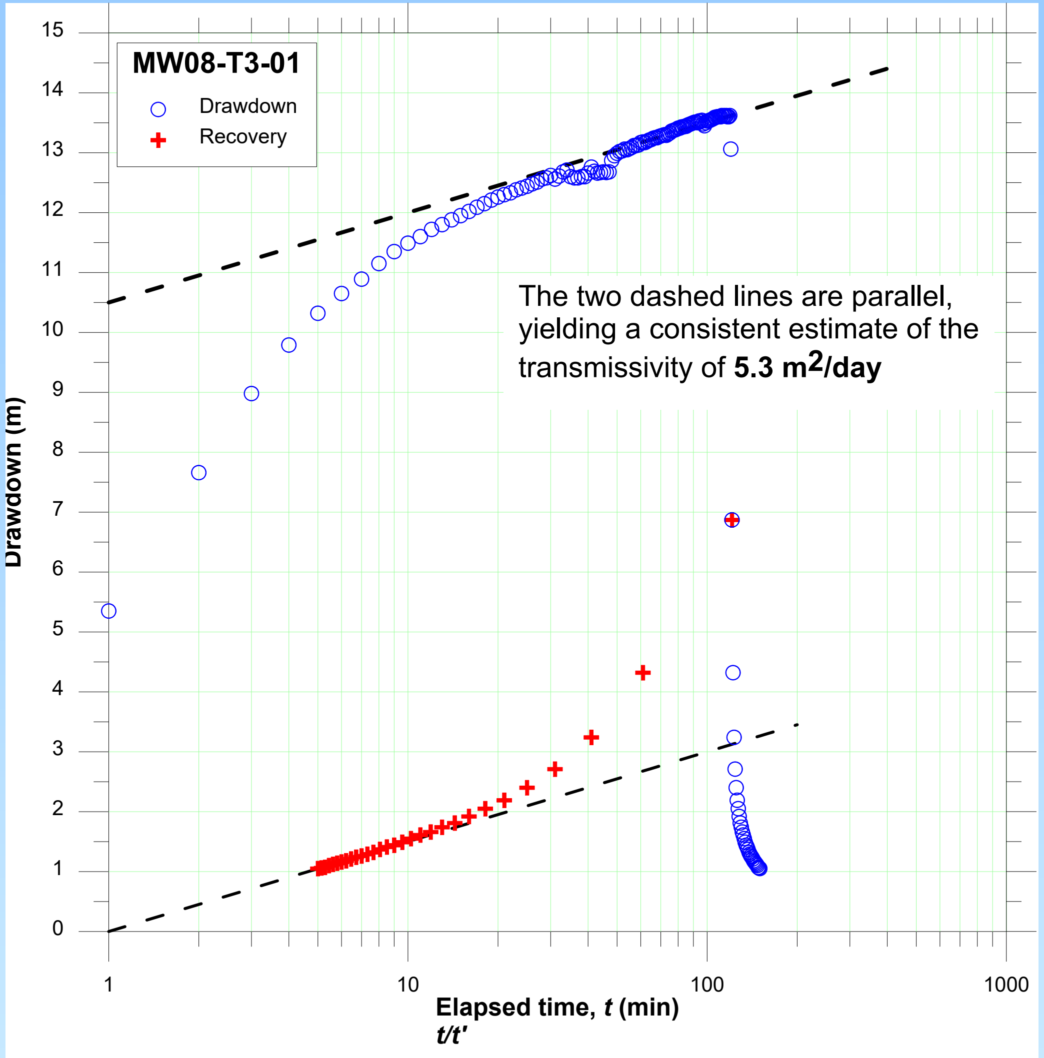
$$= \frac{Q}{4\pi T}2.303\log_{10}\left\{\frac{t}{t - t_{off}}\frac{1}{S/S'}\right\}$$

Example: Cambridge, Ontario



Applications of recovery analysis

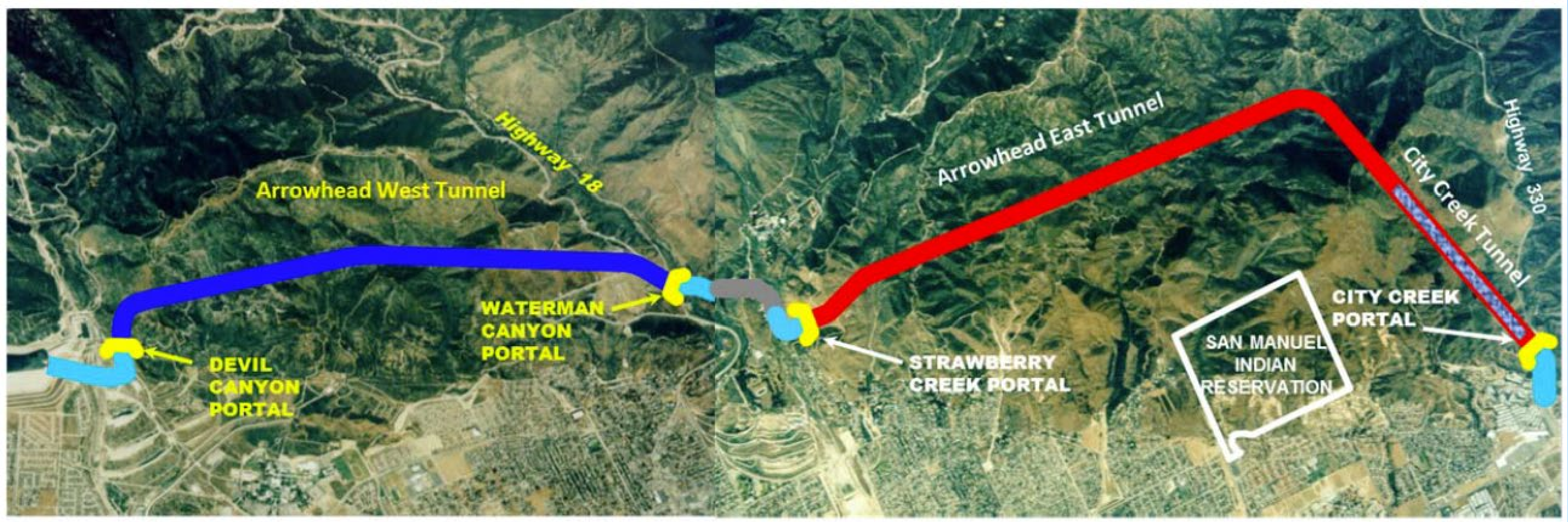
Recovery analyses may provide a useful check on the interpretations of the drawdown analyses.

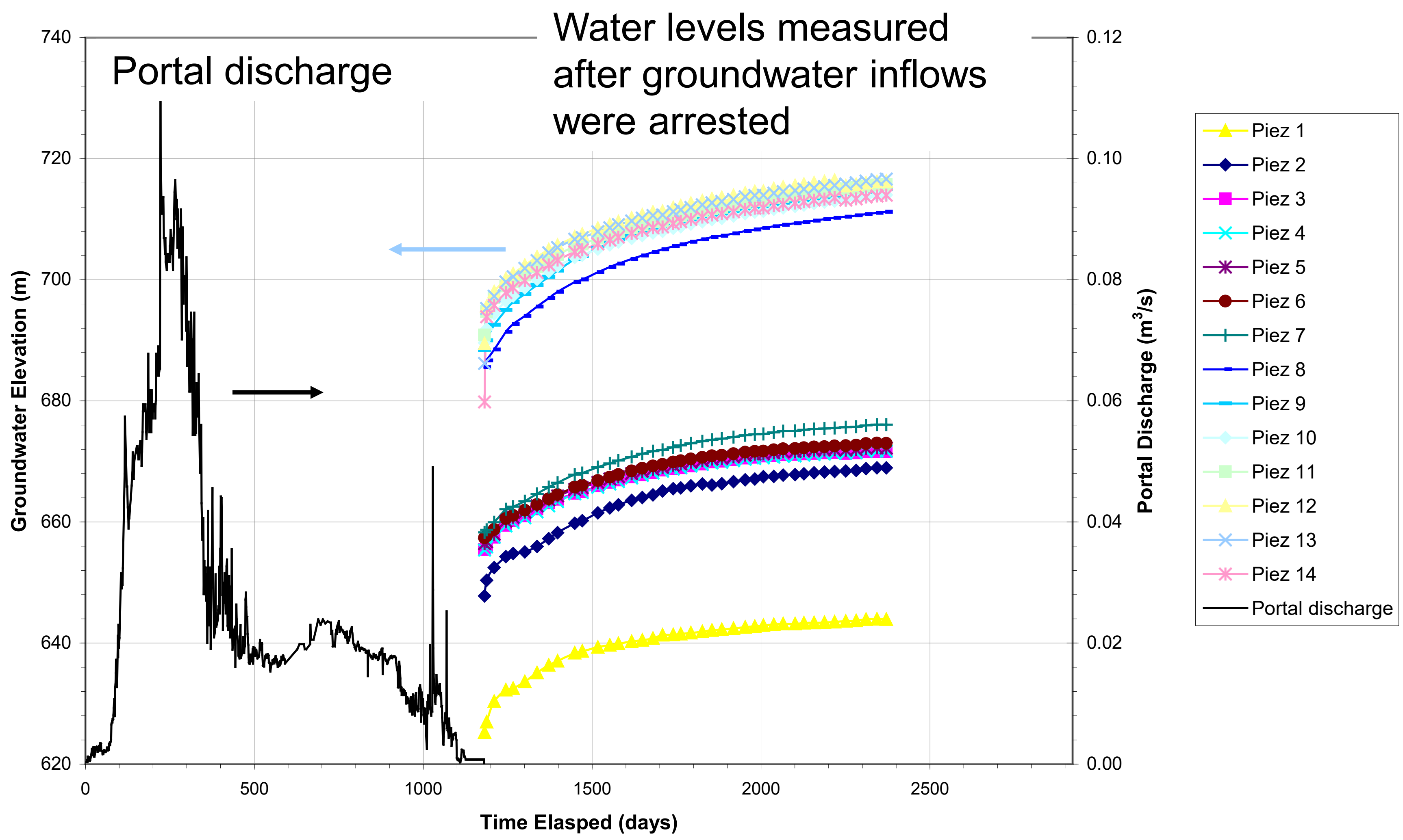


The Cooper-Jacob recovery analysis may also be useful for estimating pre-pumping conditions.

Case study:

Construction of the Arrowhead East Tunnel, San Bernardino, California





What were water levels prior to the start of tunnel construction?

Cooper-Jacob approximation:

$$s(r,t) = h_0 - h(r,t) = 2.303\frac{Q}{4\pi T}\log_{10}\left\{\frac{t}{t'}\right\}$$

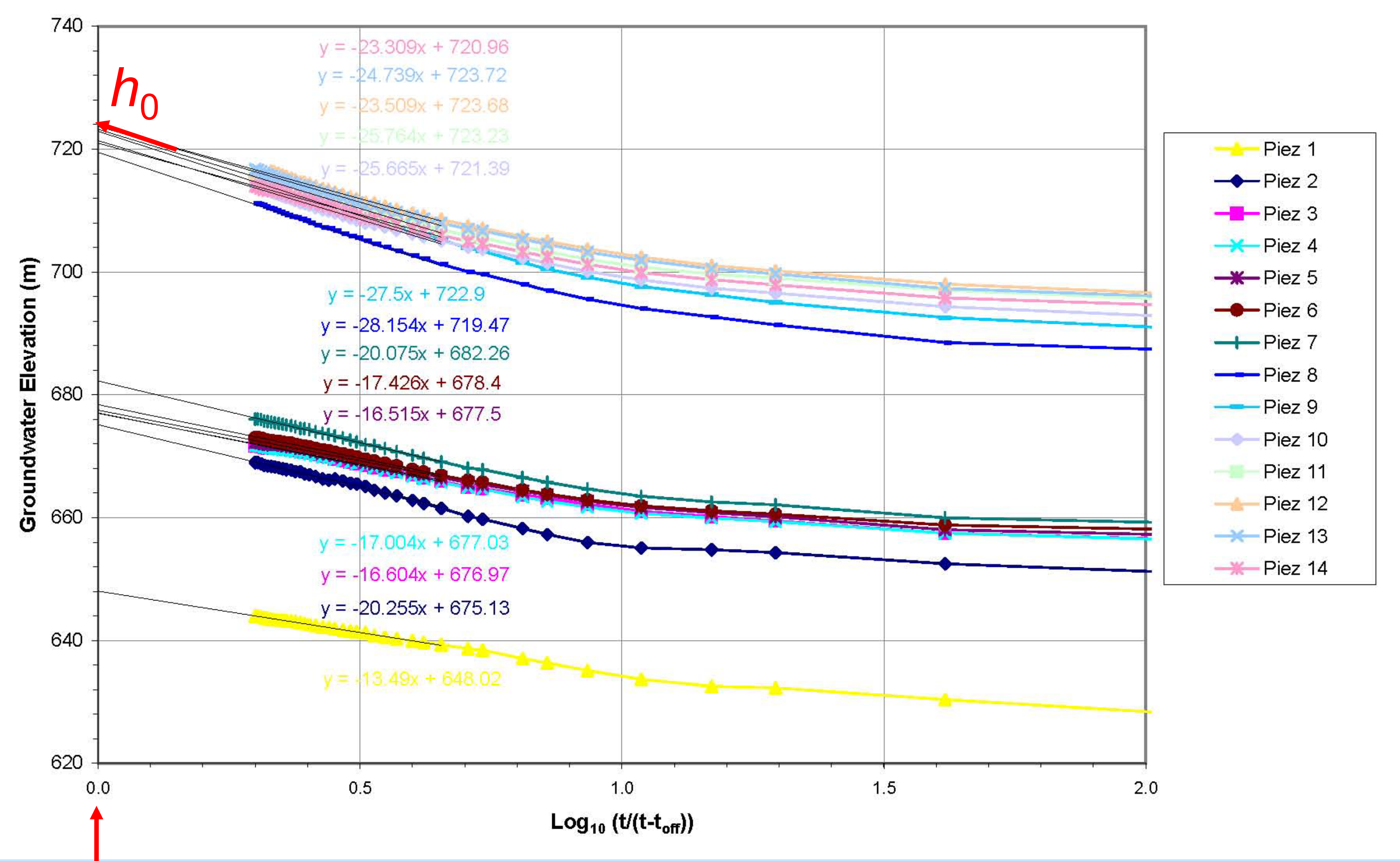
Re-arranging to solve for h_0:

$$h_0 = h(r,t) - \frac{Q}{4\pi T}2.303\log_{10}\left\{\frac{t}{t'}\right\}$$

As $\frac{t}{t'} \to 1, \quad h(r,t) \to h_0$

aka
Horner analysis

Estimation of water levels prior to the start of tunnel construction



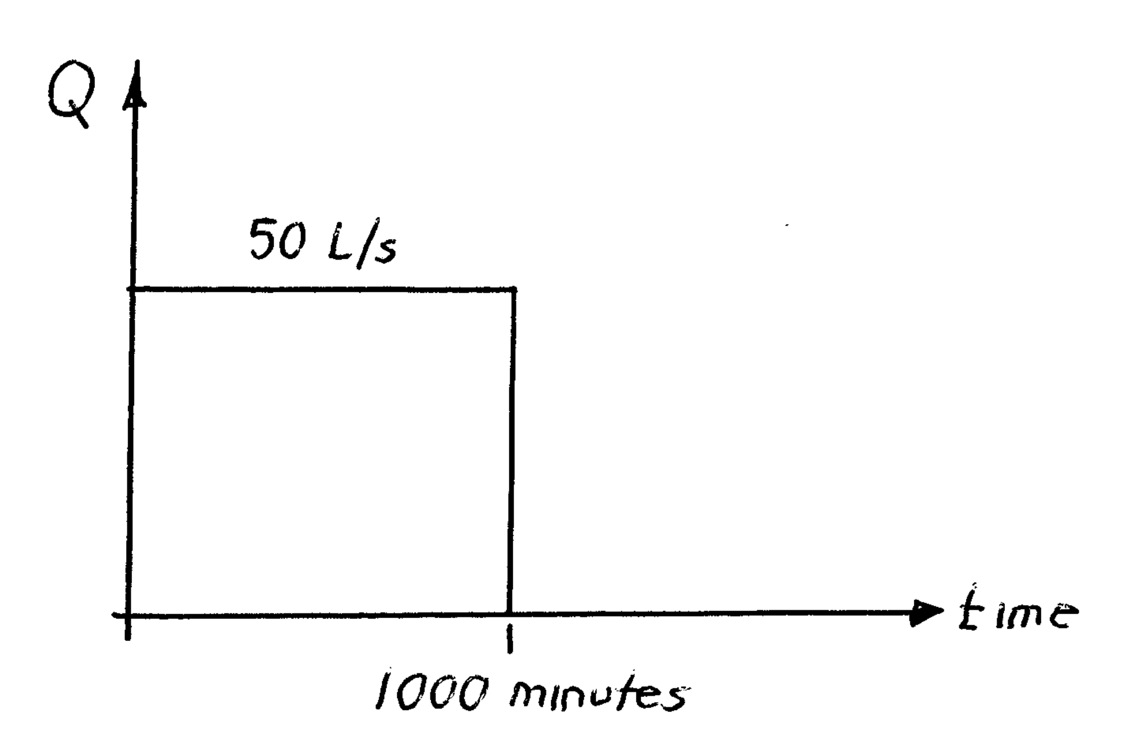
Full recovery: *t/t'* =1 (log *t/t'* = 0)

Insights from recovery data

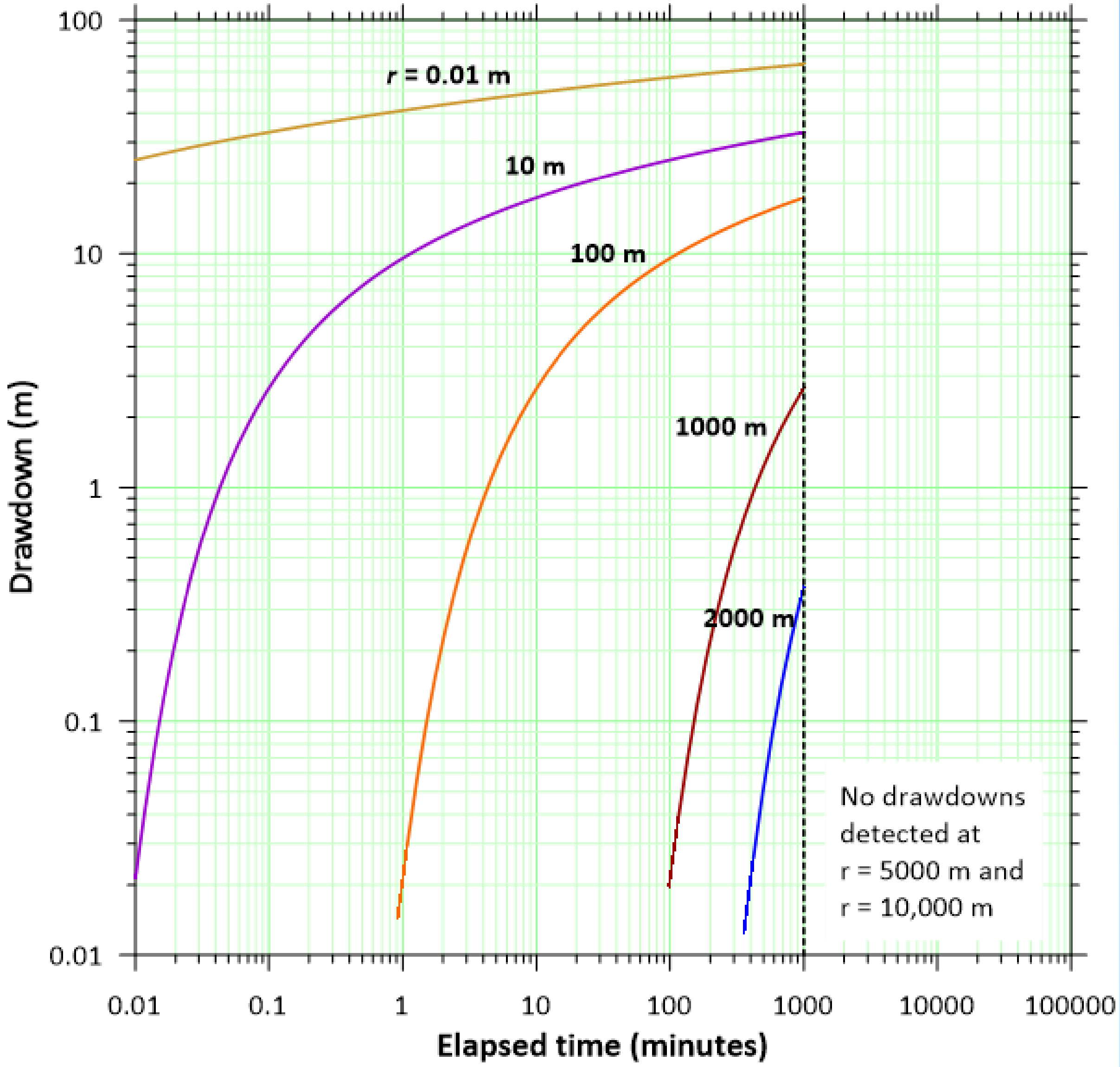
Recovery data may tell us more about our aquifer than we can glean from the drawdown data alone.

Example analysis

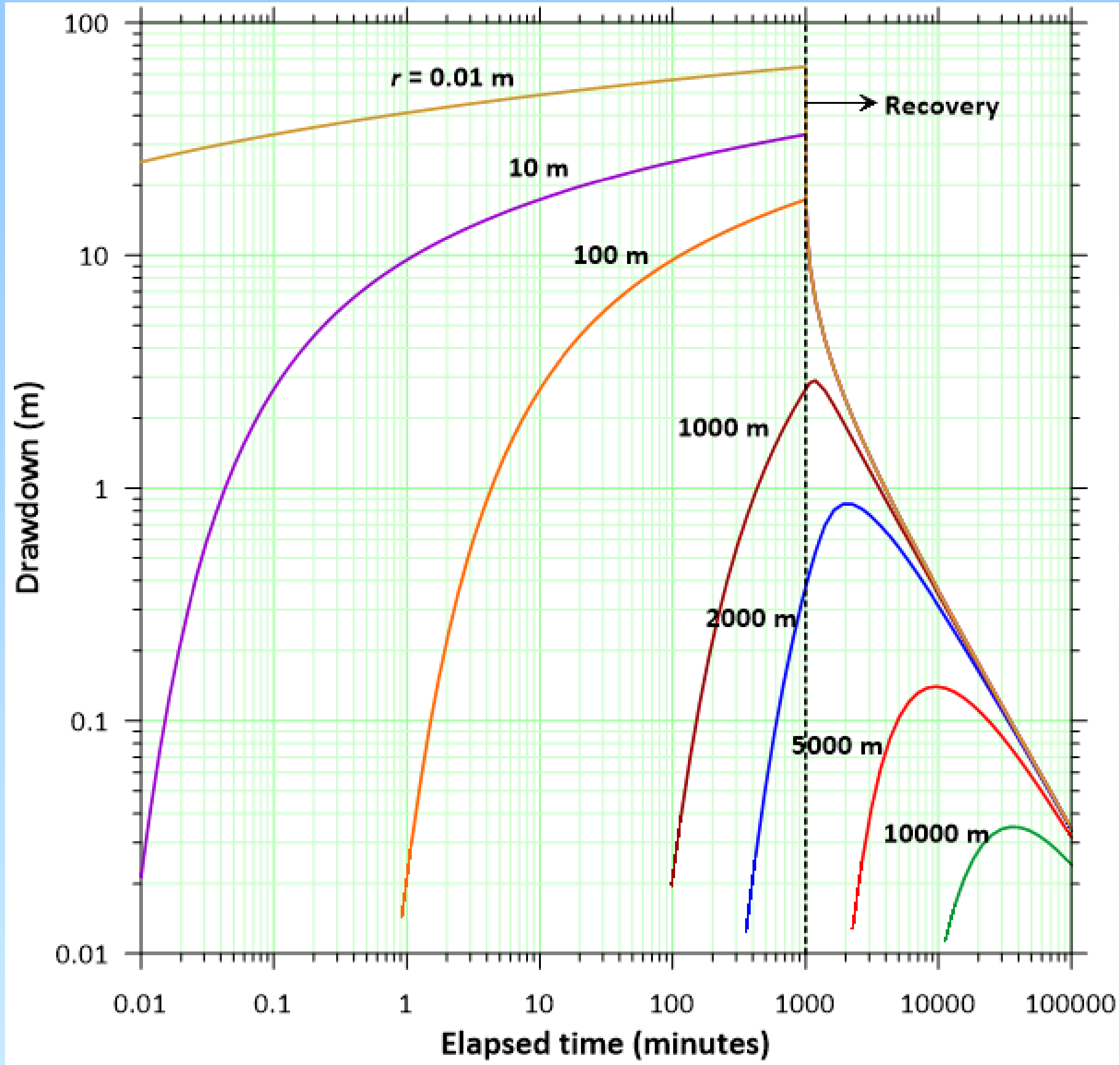
T = 100 m^2/d
S = 1.0×10^{-4}



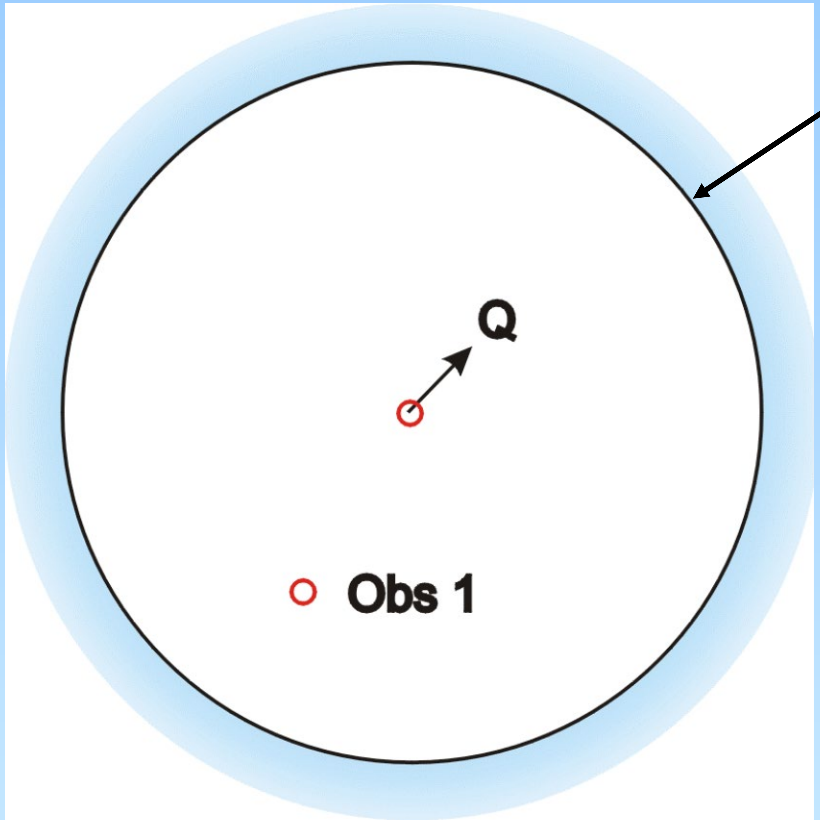
Drawdown records during pumping



Complete drawdown records



Example:
Pumping in a finite circular aquifer



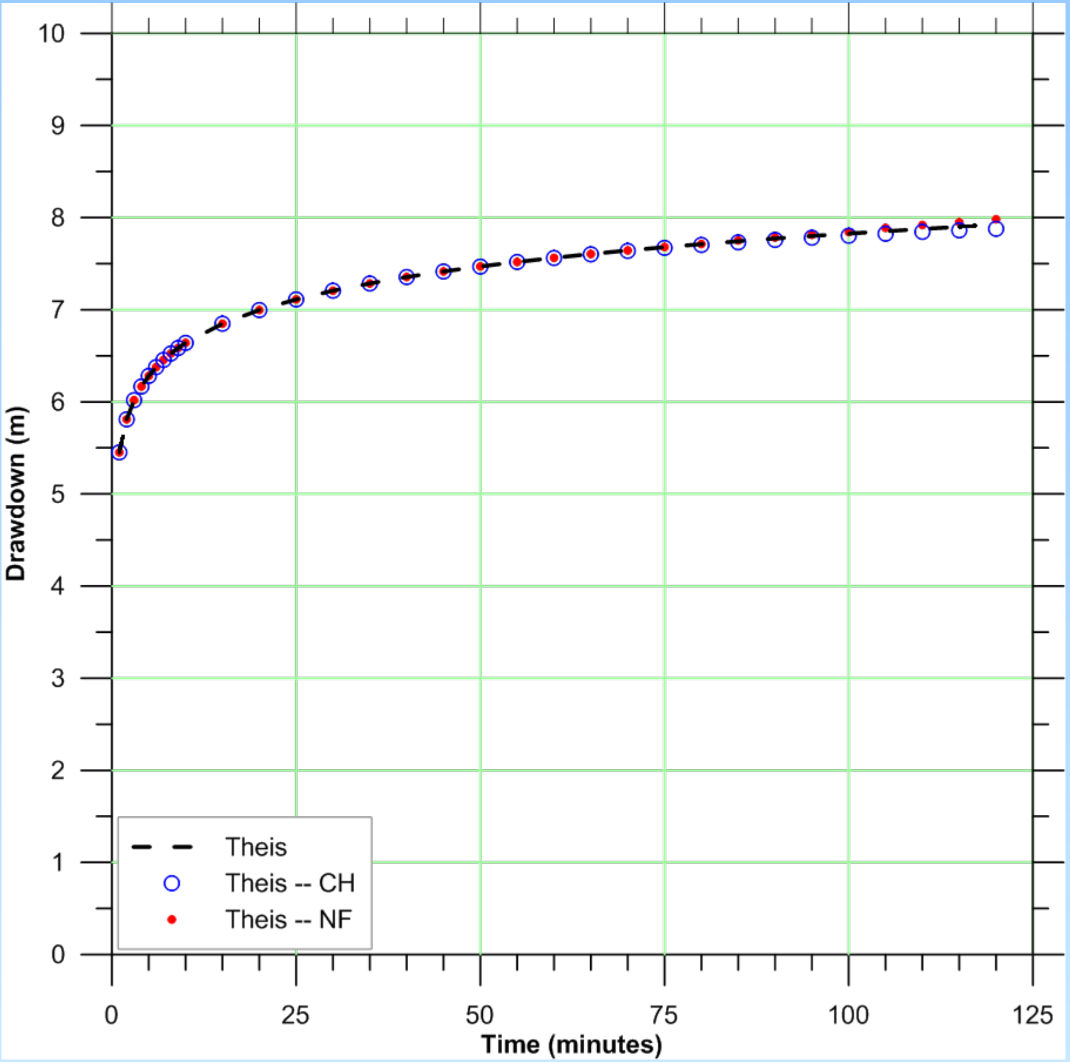
Zero-drawdown (CH)
or
Impermeable (NF)

Pumping for 120 minutes: Drawdown record

Theis: Infinite aquifer

Theis-CH: Zero-drawdown outer boundary

Theis-NF: Impermeable outer boundary

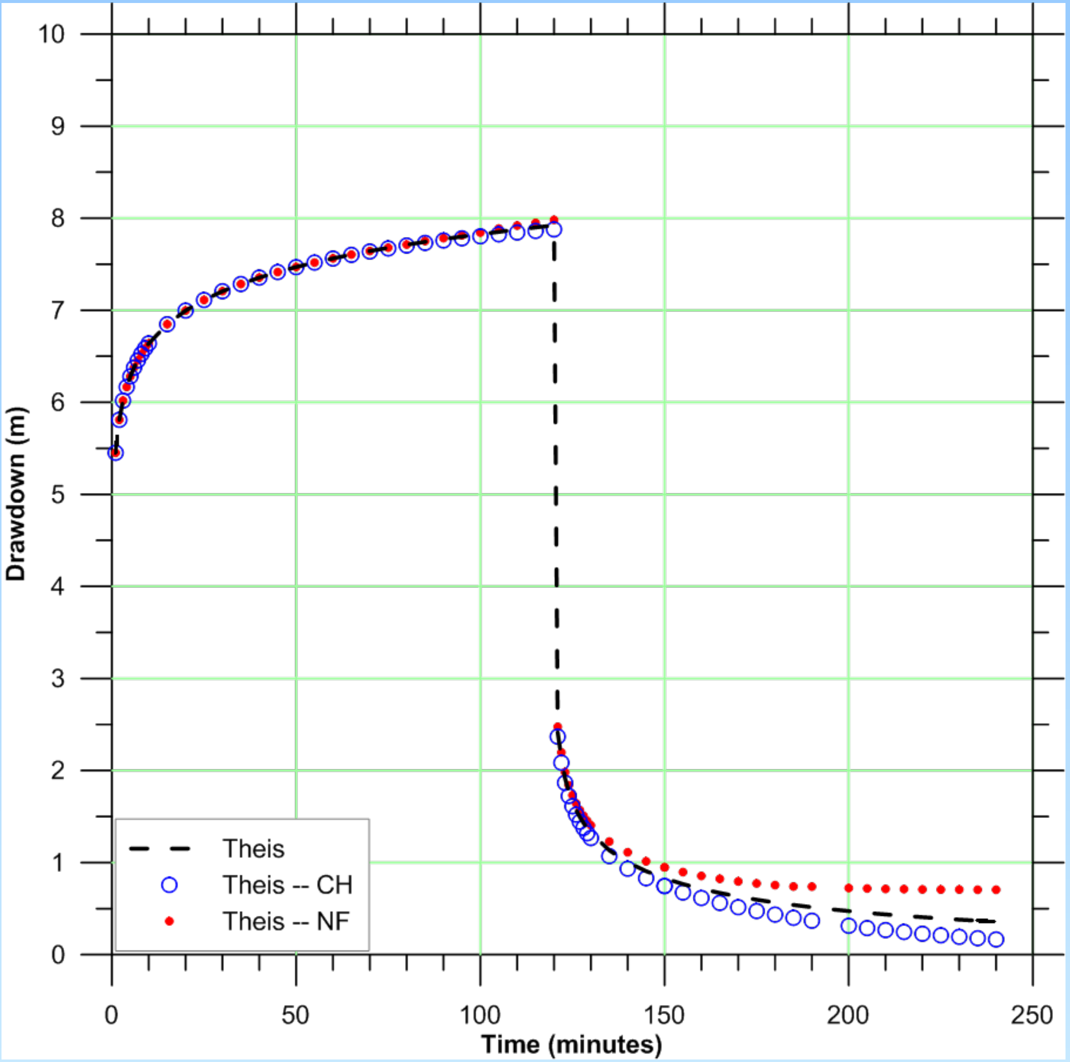


Complete drawdown and recovery record

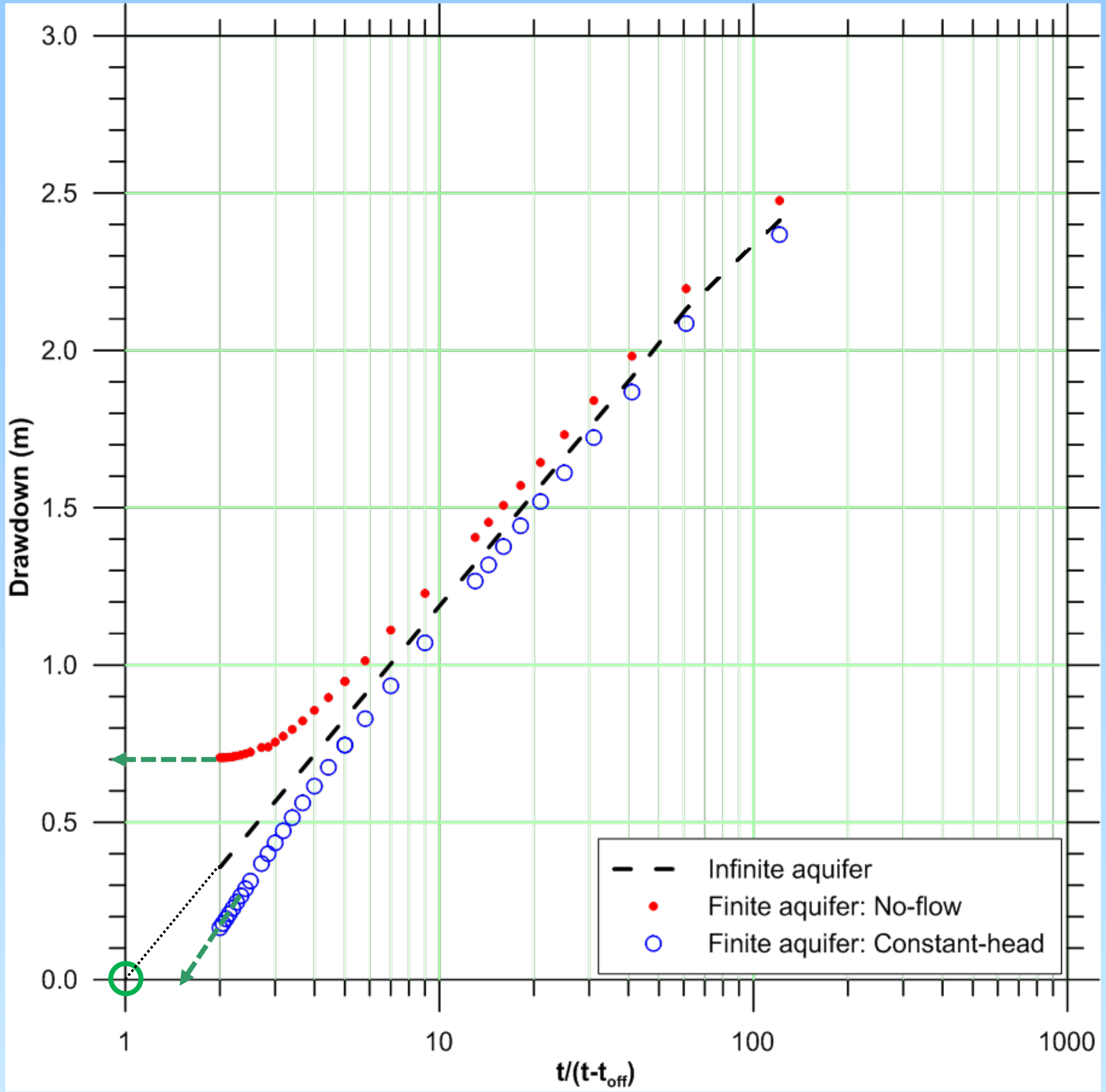
Theis: Infinite aquifer

Theis-CH: Zero-drawdown outer boundary

Theis-NF: Impermeable outer boundary



Cooper-Jacob recovery analysis



Use of recovery data to extend the effective duration of pumping

Calculation of Constant-Rate Drawdowns from Stepped-Rate Pumping Tests

by G. van der Kamp[a]

Vol. 27, No. 2—GROUND WATER—March-April 1989

Equivalent drawdown

The drawdown $s_1(r,t)$ that would have been observed if the well had remained pumping continuously at its initial rate Q_1.

General formula for a linear model

$$s(r,t) = \int_0^t Q(\tau)\ G(r, t-\tau) d\tau$$

"Impulse function"

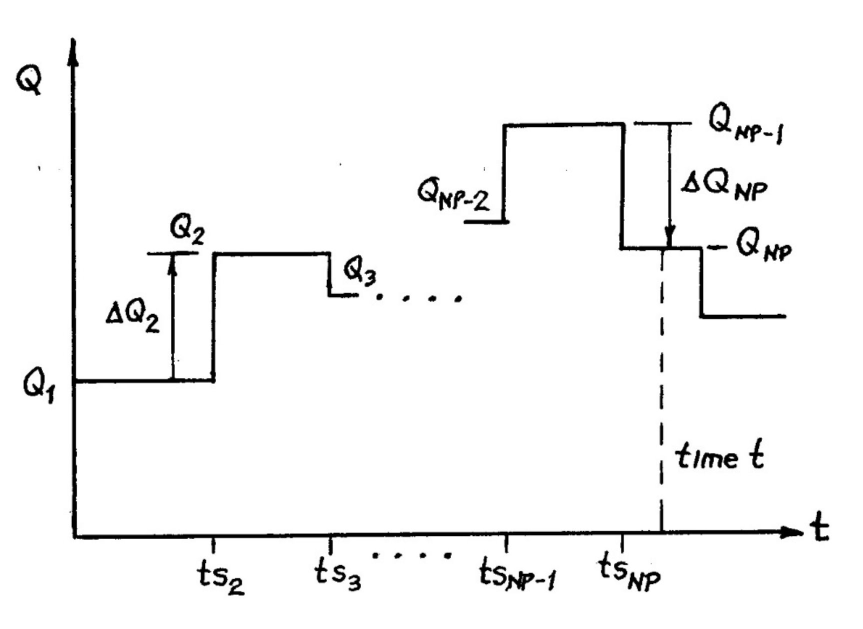
Actual pumping history

Equivalent drawdown:

$$s_1(r,t) = Q_1 \int_0^t G(\tau) d\tau$$

General formula for van der Kamp's method

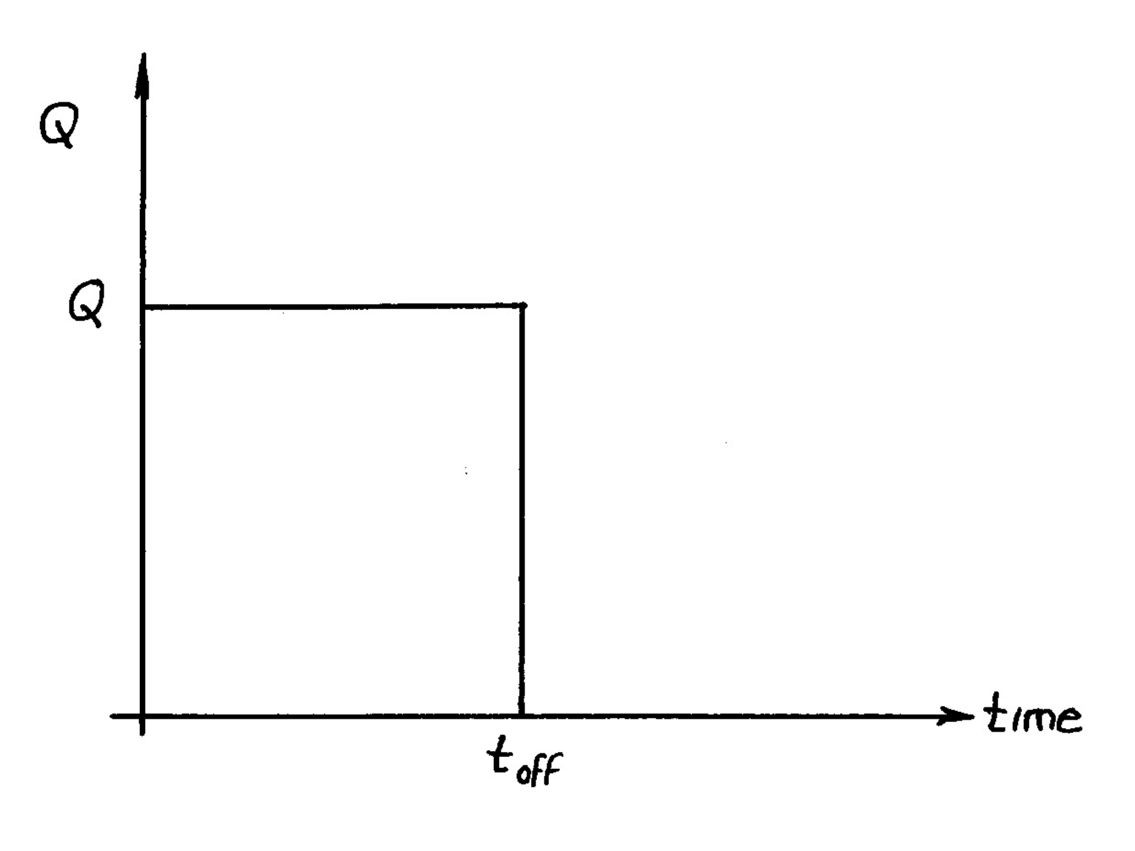
For a pumping history represented by a set of steps



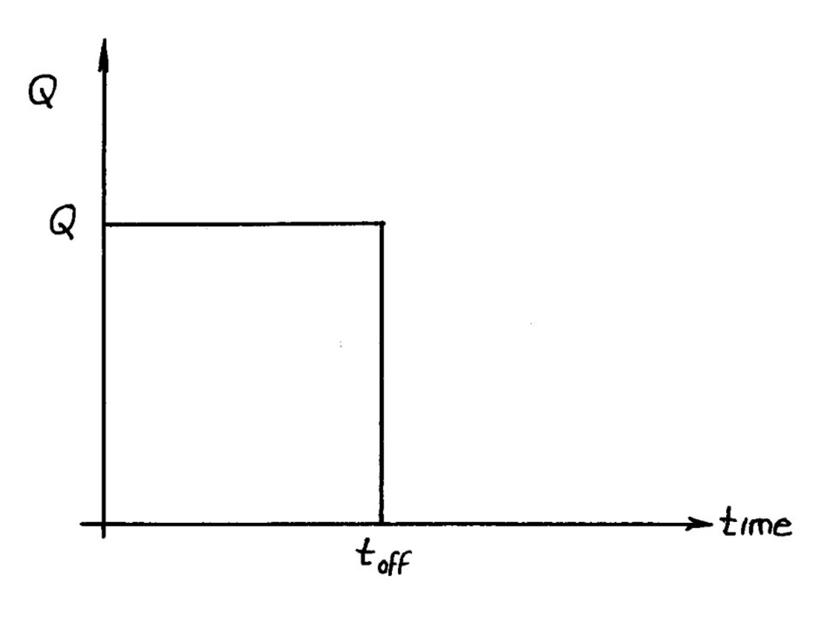
$$s_1(r,t) = s(r,t) - \frac{(Q_2 - Q_1)}{Q_1} s_1(r, t - ts_2) \cdots - \frac{(Q_{NP} - Q_{NP-1})}{Q_1} s_1(r, t - ts_{NP})$$

$s(r,t)$ ← Observed drawdowns

Pumping at a constant rate followed by recovery



In this context, the *equivalent drawdown* is that drawdown that we would have observed if the well had continued pumping.

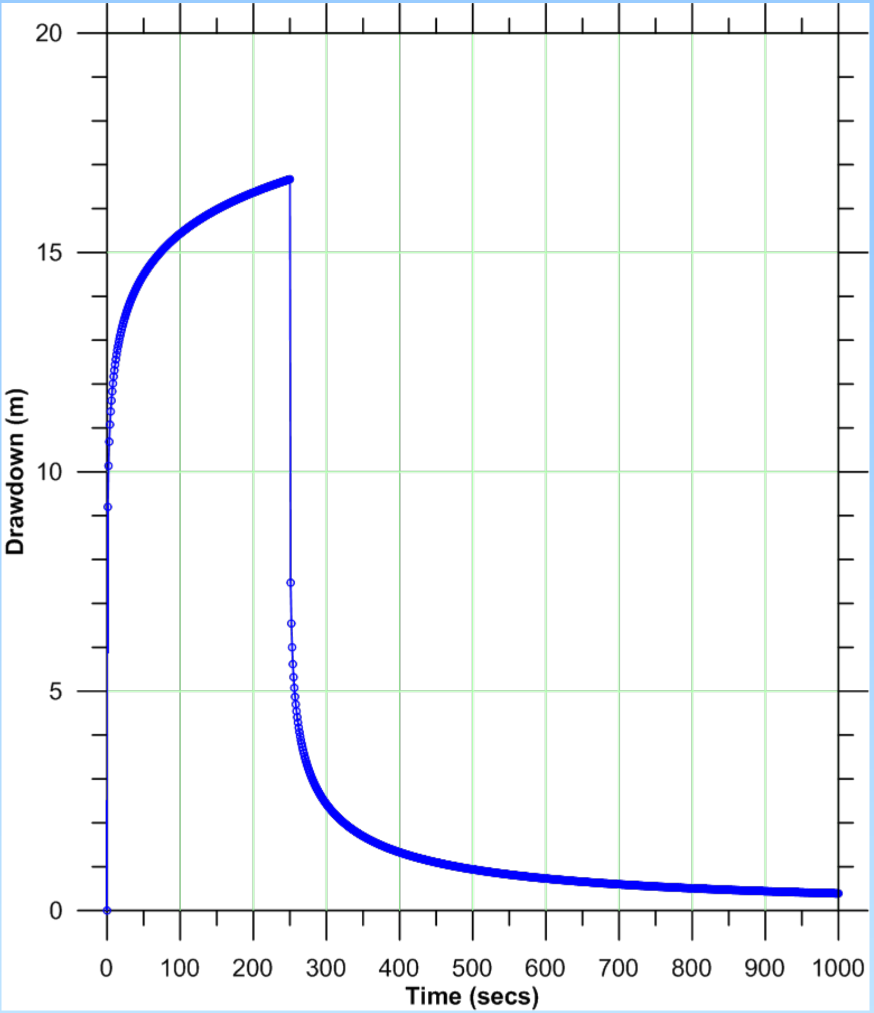


- $NP = 2$
- $ts2 = t_{off}$; $Q_2 = 0.0$

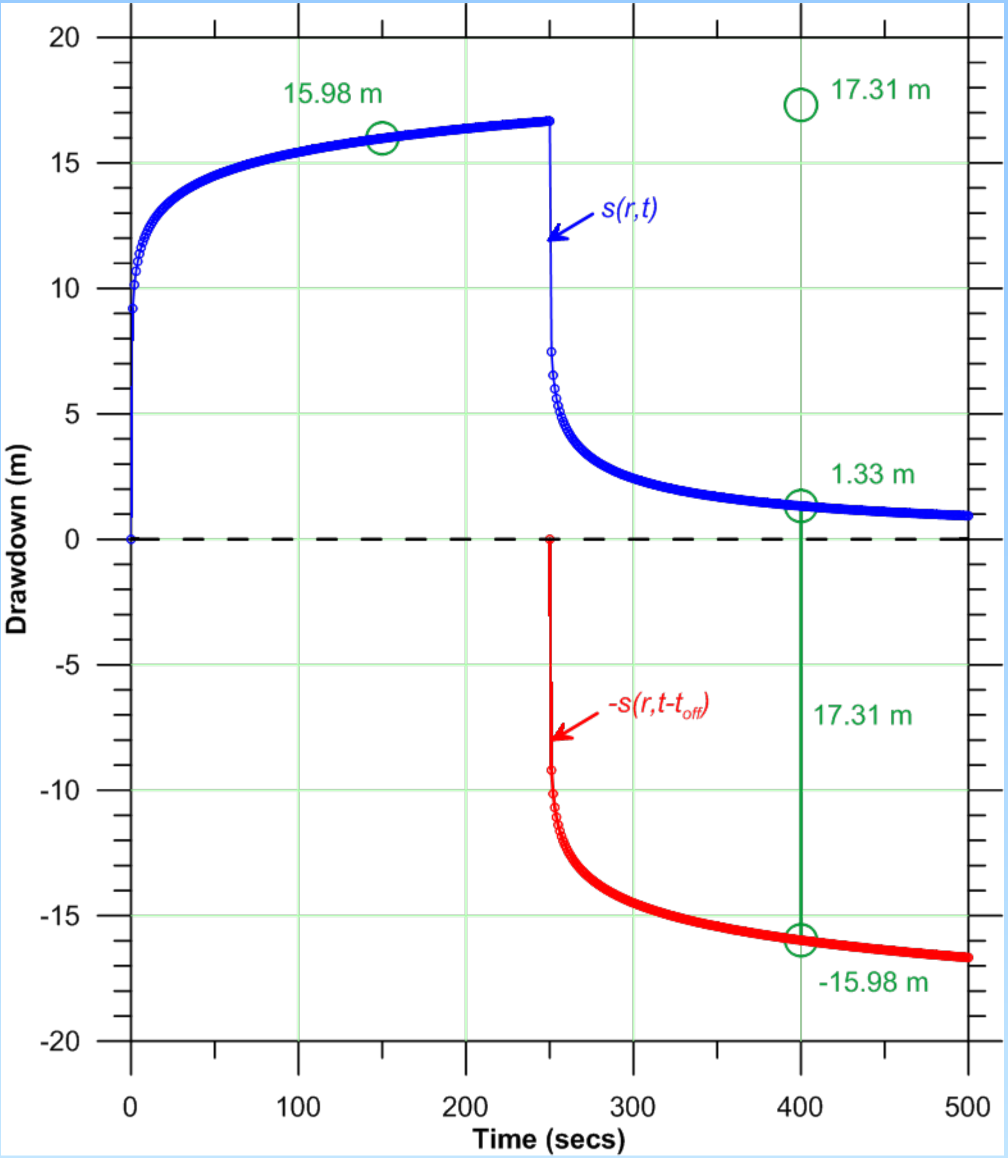
$$s_1(r,t) = s(r,t) - \frac{([0] - Q_1)}{Q_1} s_1\left(r, t - \left[t_{off}\right]\right)$$

$$\rightarrow s_1(r,t) = s(r,t) + s_1\left(r, t - t_{off}\right)$$

Example



Pumping stopped after 250 seconds.
What drawdown would we have observed at 400 seconds if the well had continued pumping?



$$s_1(r,t) = s(r,t) + s_1(r,t-t_{off})$$

@ t = 400 s

s(400 s) = 1.33 m

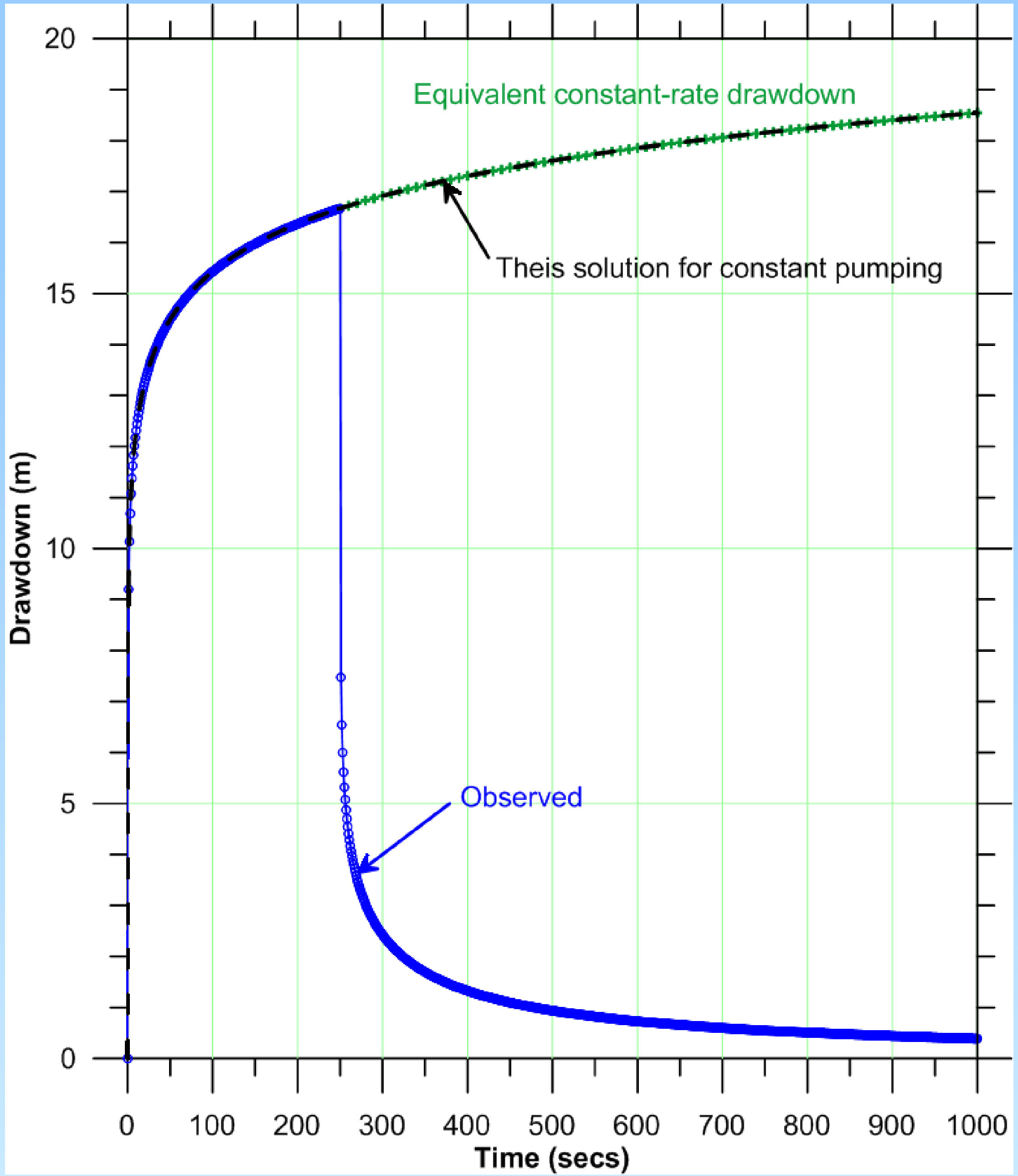
t-t_{off} = 400 – 250 =150 s

s(150 s) = 15.98 m

s_1(400 s) = 1.33 m

+15.98 m

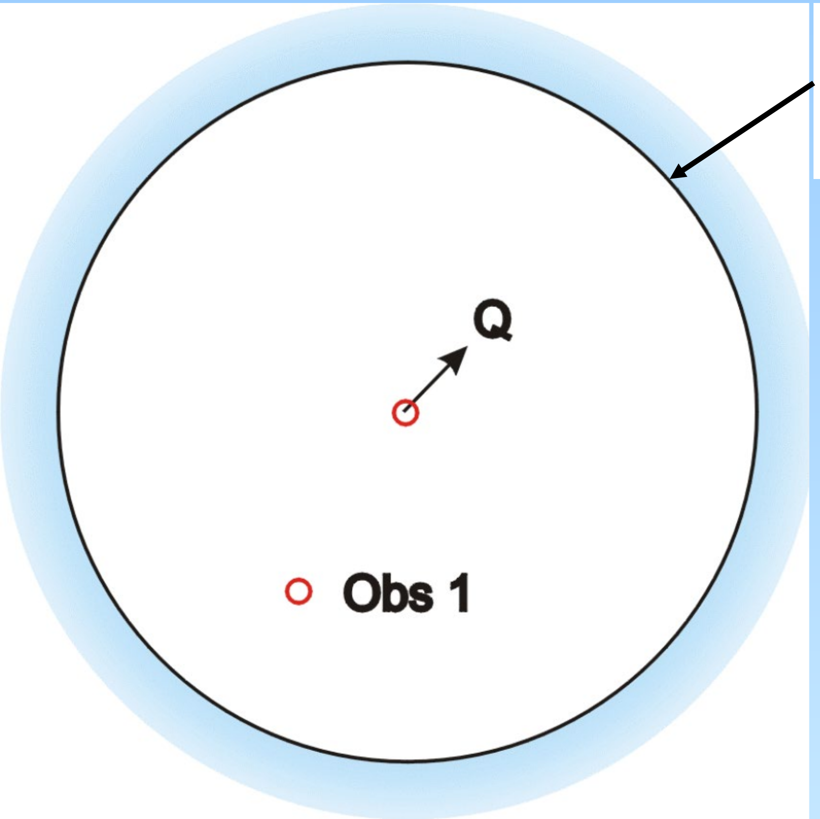
= 17.31 m



Equivalent constant-rate drawdown
Theis solution for constant pumping
Observed
Drawdown (m)
Time (secs)
0
5
10
15
20
0
100
200
300
400
500
600
700
800
900
1000

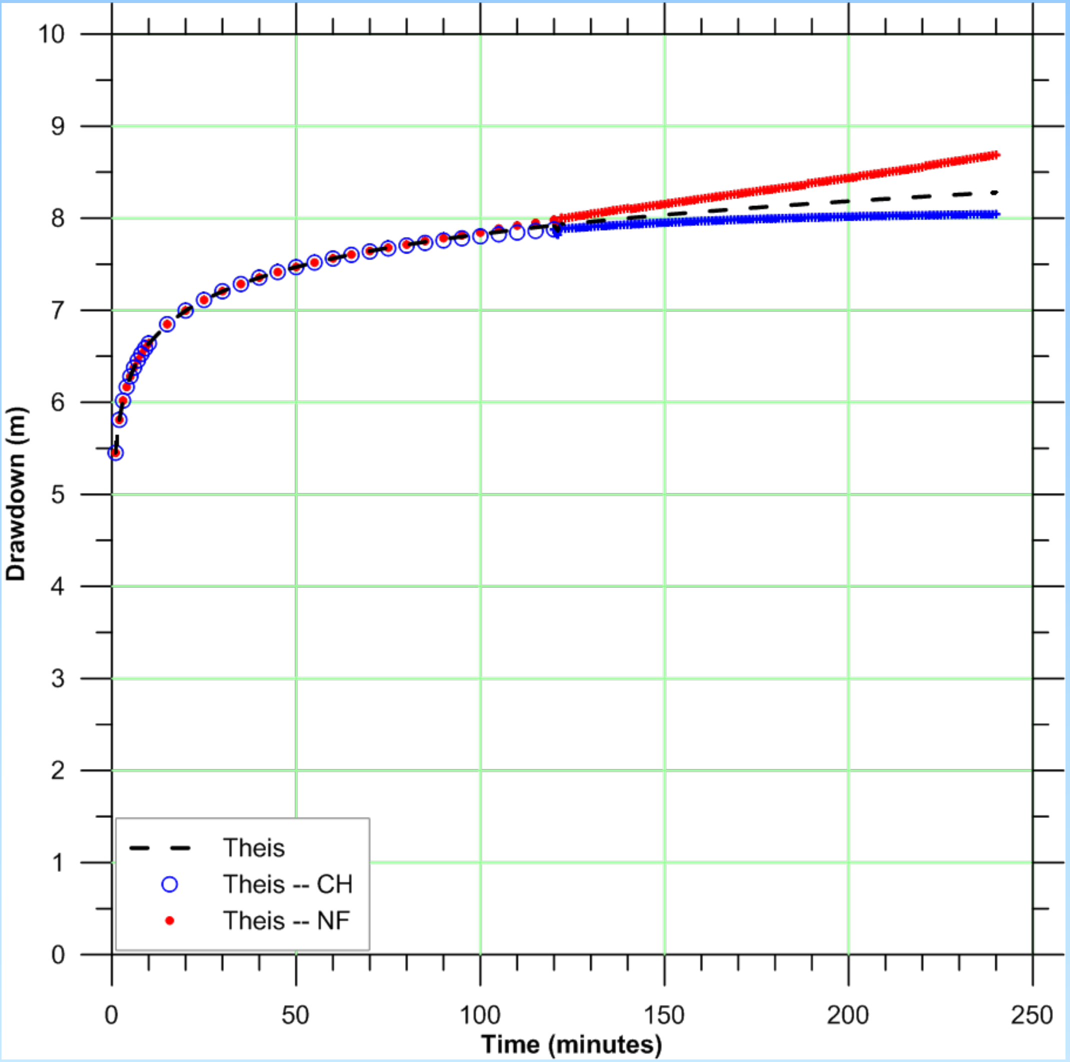
Example revisited:

Pumping in a finite circular aquifer

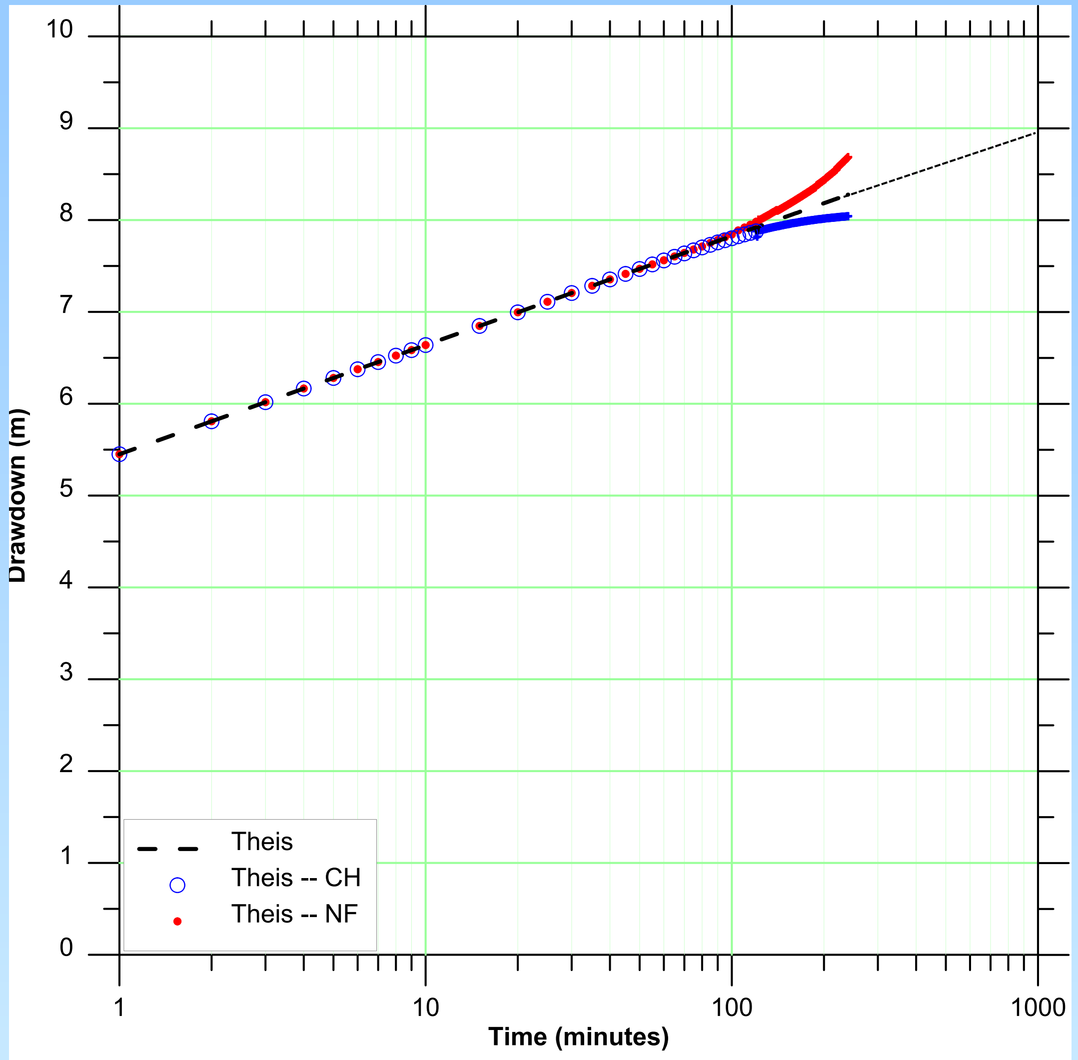


Zero-drawdown (CH)
Impermeable (NF)

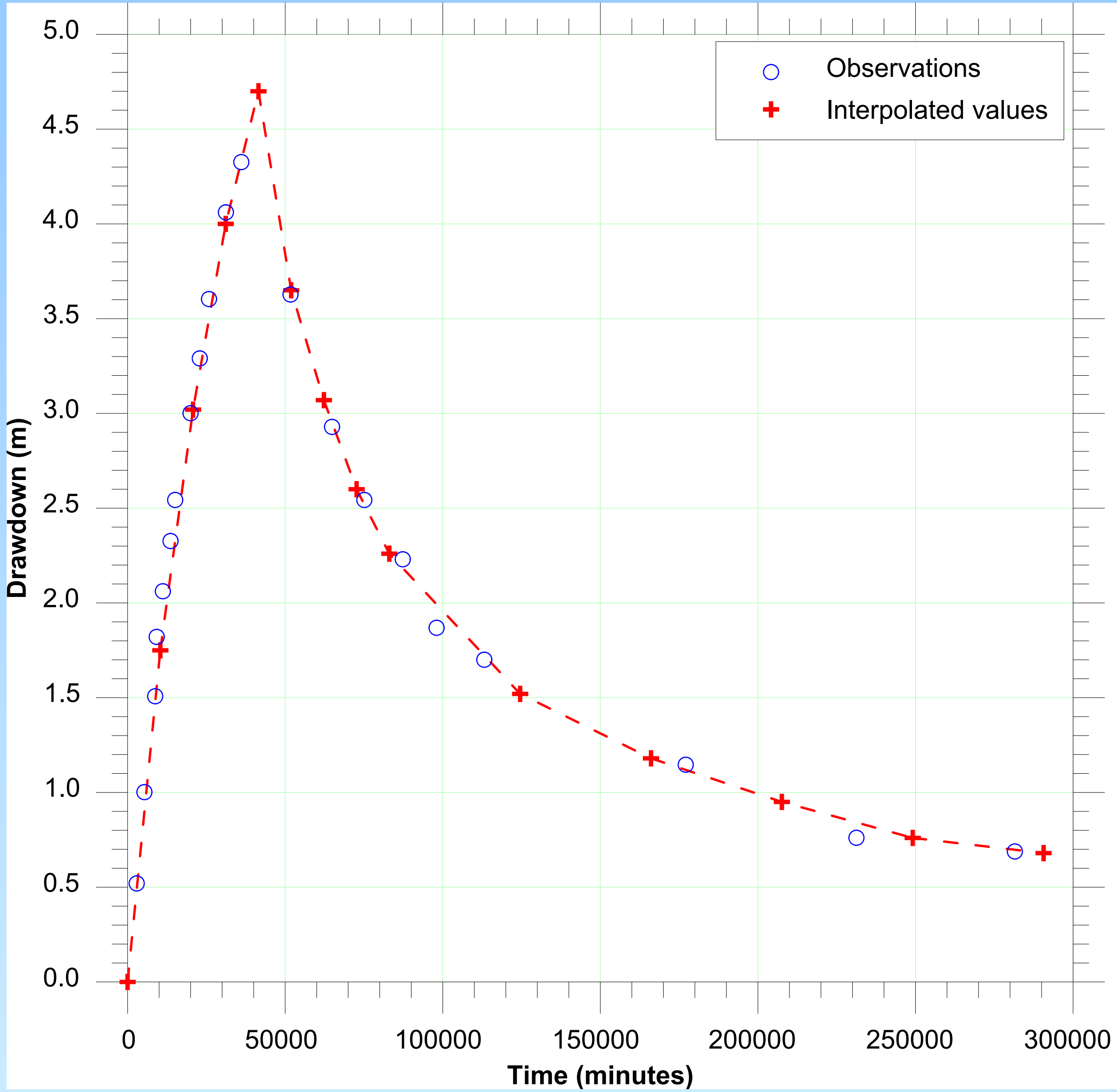
van der Kamp
Equivalent constant-rate drawdowns



Drawdown record: Semilog plot with the extended drawdown record



Case study:
Estevan, Saskatchewan (1984)



t (minutes)	s (t) (m)	$t-t_{off}$ (minutes)	$s(t-t_{off})$ (m)	s_1 (m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07			
72660	2.60			
83040	2.26			
124560	1.52			
166080	1.18			
207600	0.95			
249120	0.76			
290640	0.68			

t (minutes)	s (t) (m)	t-t_{off} (minutes)	s (t-t_{off}) (m)	s_1 (m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60			
83040	2.26			
124560	1.52			
166080	1.18			
207600	0.95			
249120	0.76			
290640	0.68			

t (minutes)	$s(t)$ (m)	$t-t_{off}$ (minutes)	$s(t-t_{off})$ (m)	s_1 (m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60	31140	4.00	6.60
83040	2.26			
124560	1.52			
166080	1.18			
207600	0.95			
249120	0.76			
290640	0.68			

t (minutes)	s (t) (m)	$t-t_{off}$ (minutes)	$s(t-t_{off})$ (m)	s_1 (m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60	31140	4.00	6.60
83040	2.26	41520	4.70	6.96
124560	1.52			
166080	1.18			
207600	0.95			
249120	0.76			
290640	0.68			

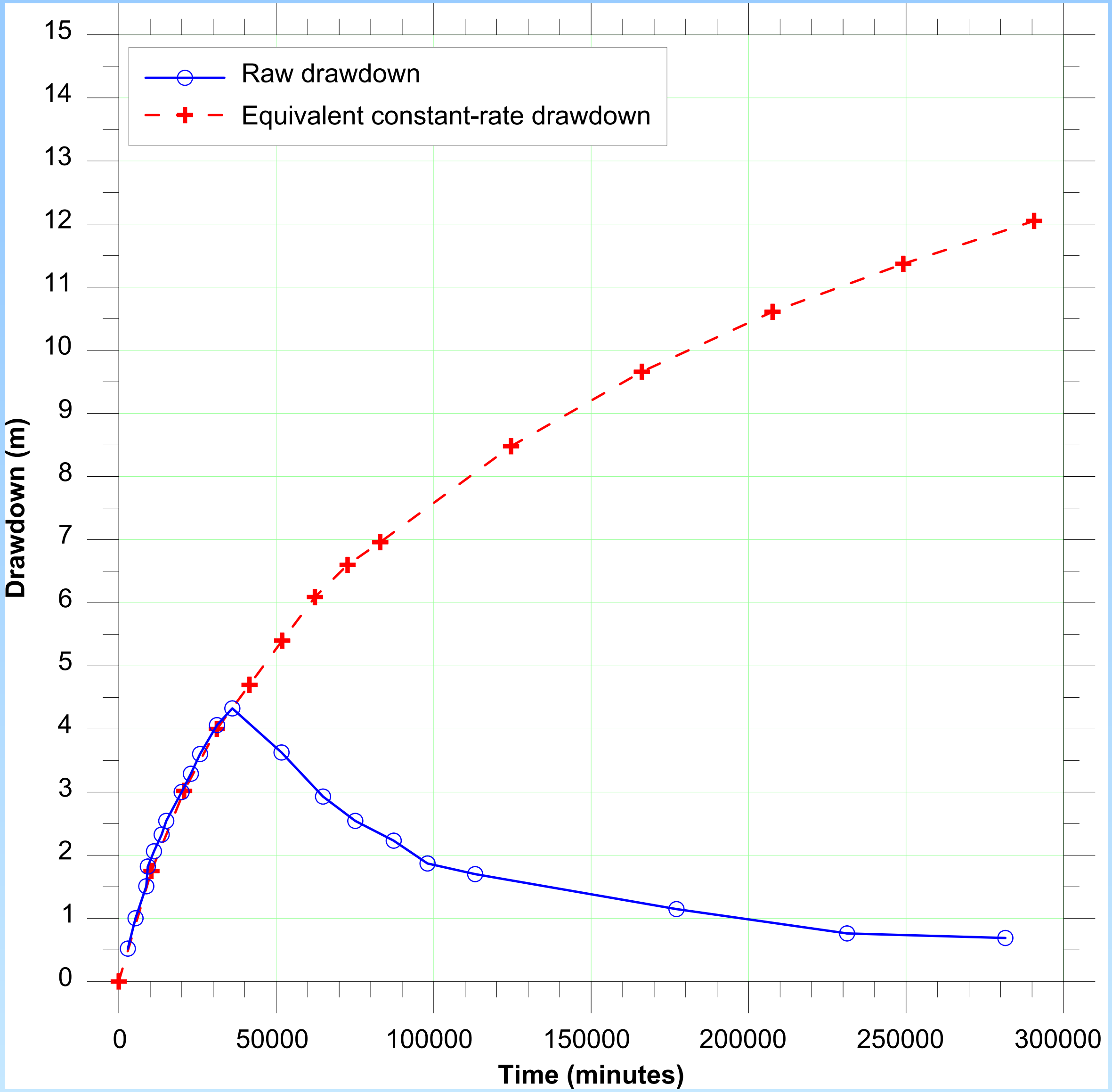
t (minutes)	s (t) (m)	t-t_{off} (minutes)	s (t-t_{off}) (m)	s_1 (m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60	31140	4.00	6.60
83040	2.26	41520	4.70	6.96
124560	1.52	83040	6.96	8.48
166080	1.18			
207600	0.95			
249120	0.76			
290640	0.68			

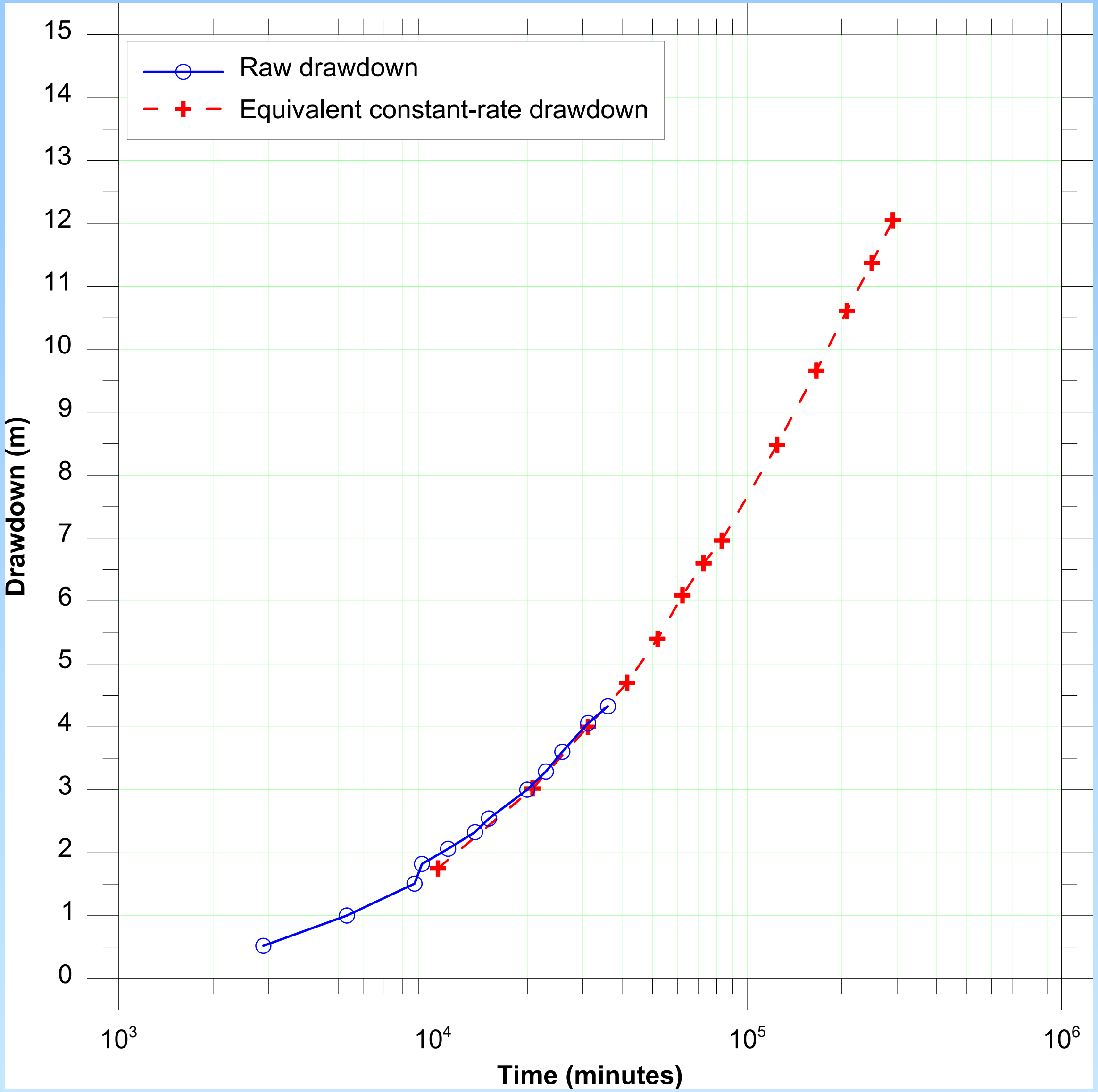
t (minutes)	s (t) (m)	$t\text{-}t_{off}$ (minutes)	s $(t\text{-}t_{off})$ (m)	s_1 (m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60	31140	4.00	6.60
83040	2.26	41520	4.70	6.96
124560	1.52	83040	6.96	8.48
166080	1.18	124560	8.48	9.66
207600	0.95			
249120	0.76			
290640	0.68			

t (minutes)	s (t) (m)	$t-t_{off}$ (minutes)	$s(t-t_{off})$ (m)	s_1 (m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60	31140	4.00	6.60
83040	2.26	41520	4.70	6.96
124560	1.52	83040	6.96	8.48
166080	1.18	124560	8.48	9.66
207600	0.95	166080	9.66	10.61
249120	0.76			
290640	0.68			

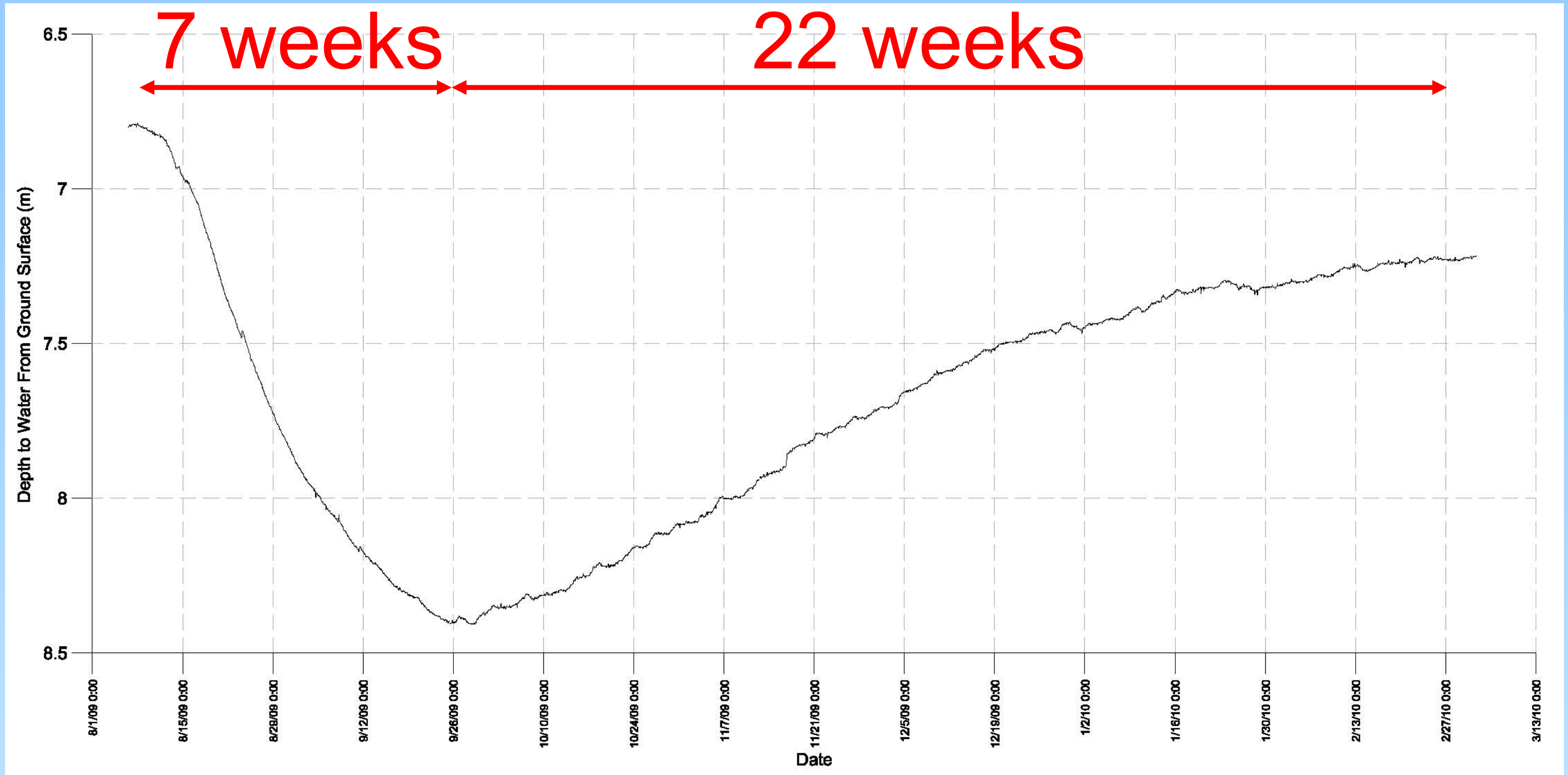
t (minutes)	$s(t)$ (m)	$t-t_{off}$ (minutes)	$s(t-t_{off})$ (m)	s_1 (m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60	31140	4.00	6.60
83040	2.26	41520	4.70	6.96
124560	1.52	83040	6.96	8.48
166080	1.18	124560	8.48	9.66
207600	0.95	166080	9.66	10.61
249120	0.76	207600	10.61	11.37
290640	0.68			

t (minutes)	s (t) (m)	$t-t_{off}$ (minutes)	$s(t-t_{off})$ (m)	s_1 (m)
0	0.00			0.00
10380	1.75			1.75
20760	3.02			3.02
31140	4.00			4.00
41520	4.70	0		4.70
51900	3.65	10380	1.75	5.40
62280	3.07	20760	3.02	6.09
72660	2.60	31140	4.00	6.60
83040	2.26	41520	4.70	6.96
124560	1.52	83040	6.96	8.48
166080	1.18	124560	8.48	9.66
207600	0.95	166080	9.66	10.61
249120	0.76	207600	10.61	11.37
290640	0.68	249120	11.37	12.05

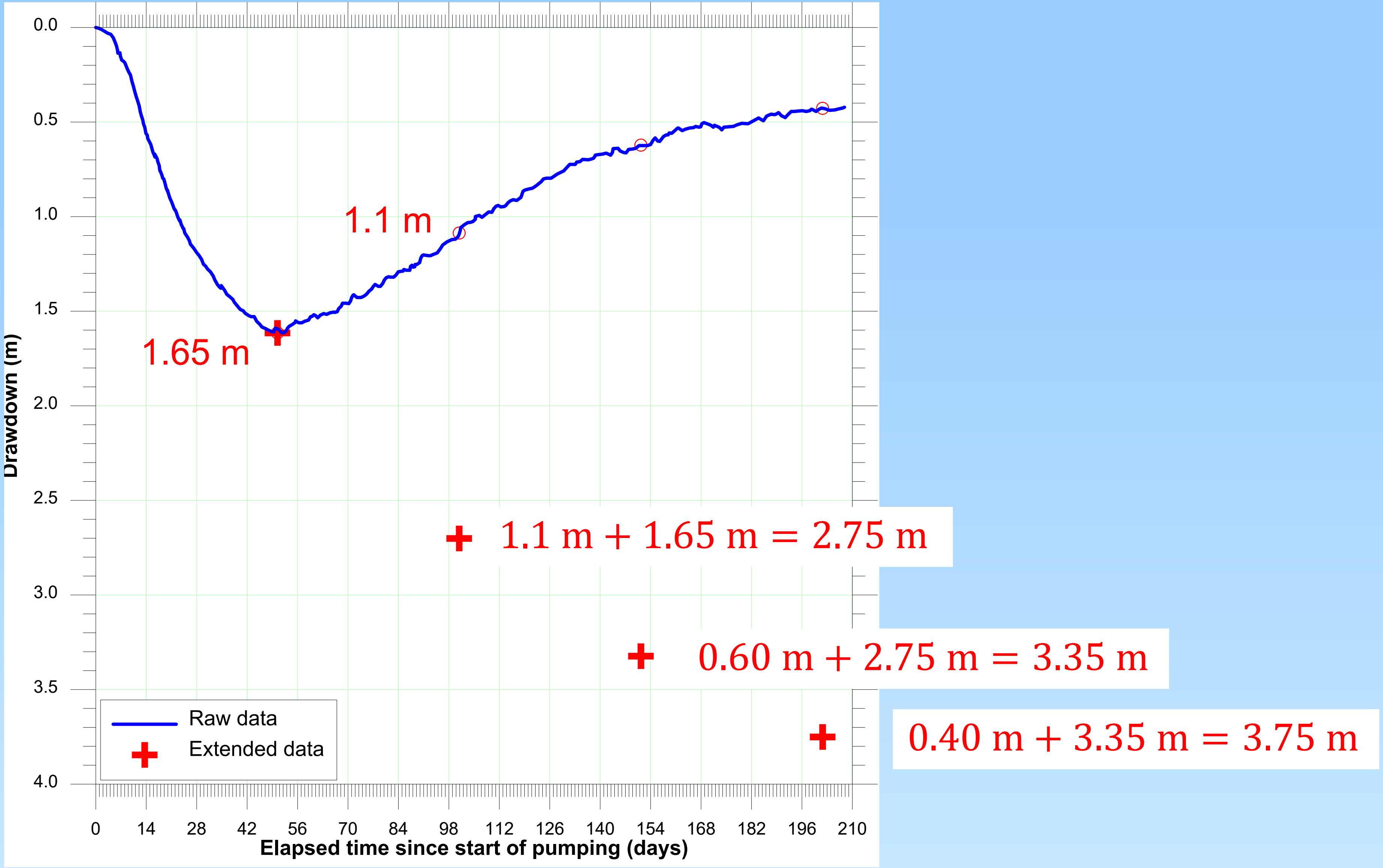

Raw drawdown
Equivalent constant-rate drawdown
Drawdown (m)
Time (minutes)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
0
50000
100000
150000
200000
250000
300000

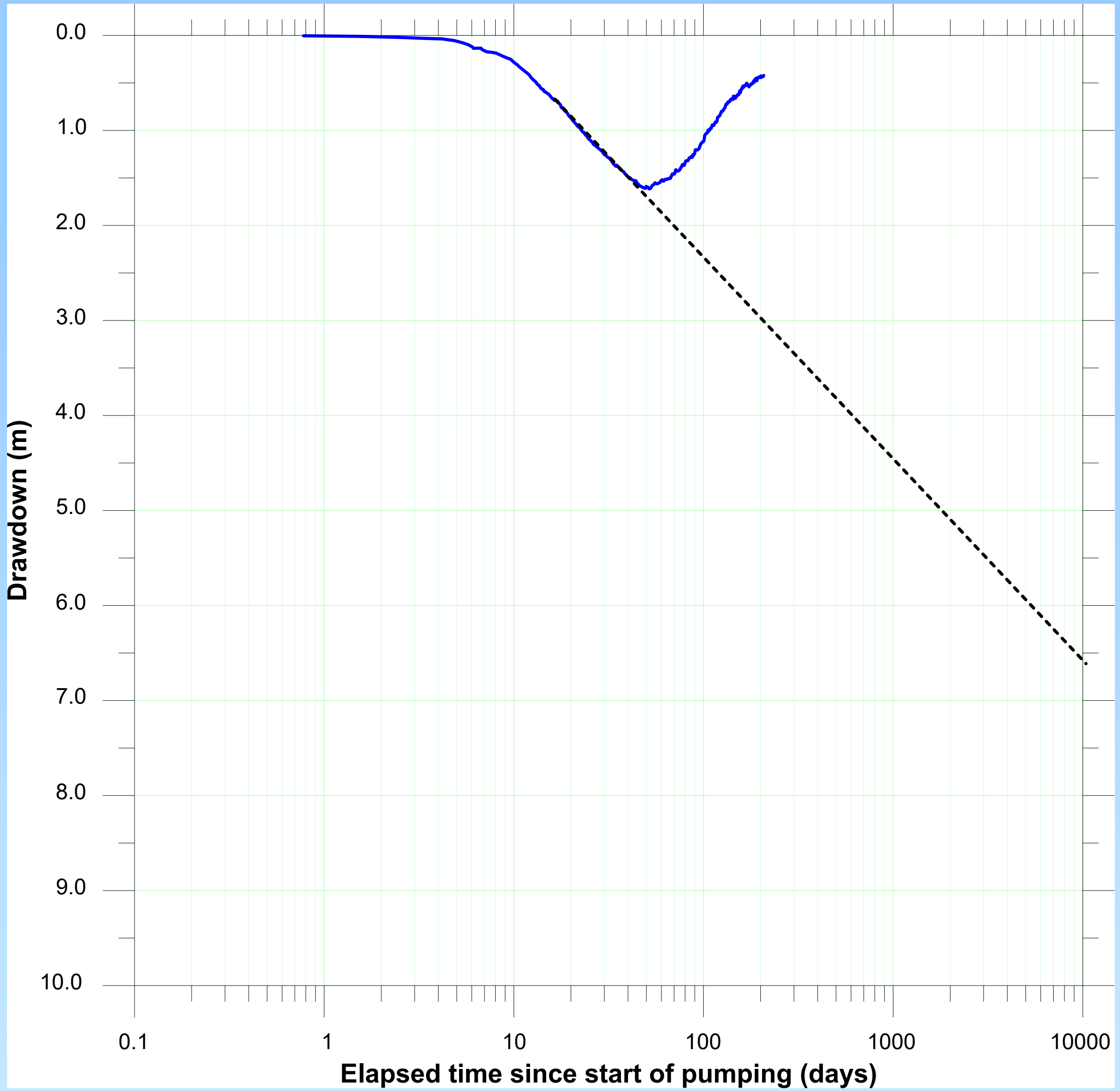

Raw drawdown
Equivalent constant-rate drawdown
Drawdown (m)
Time (minutes)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
10^3
10^4
10^5
10^6

Case study: MW-1 observation well

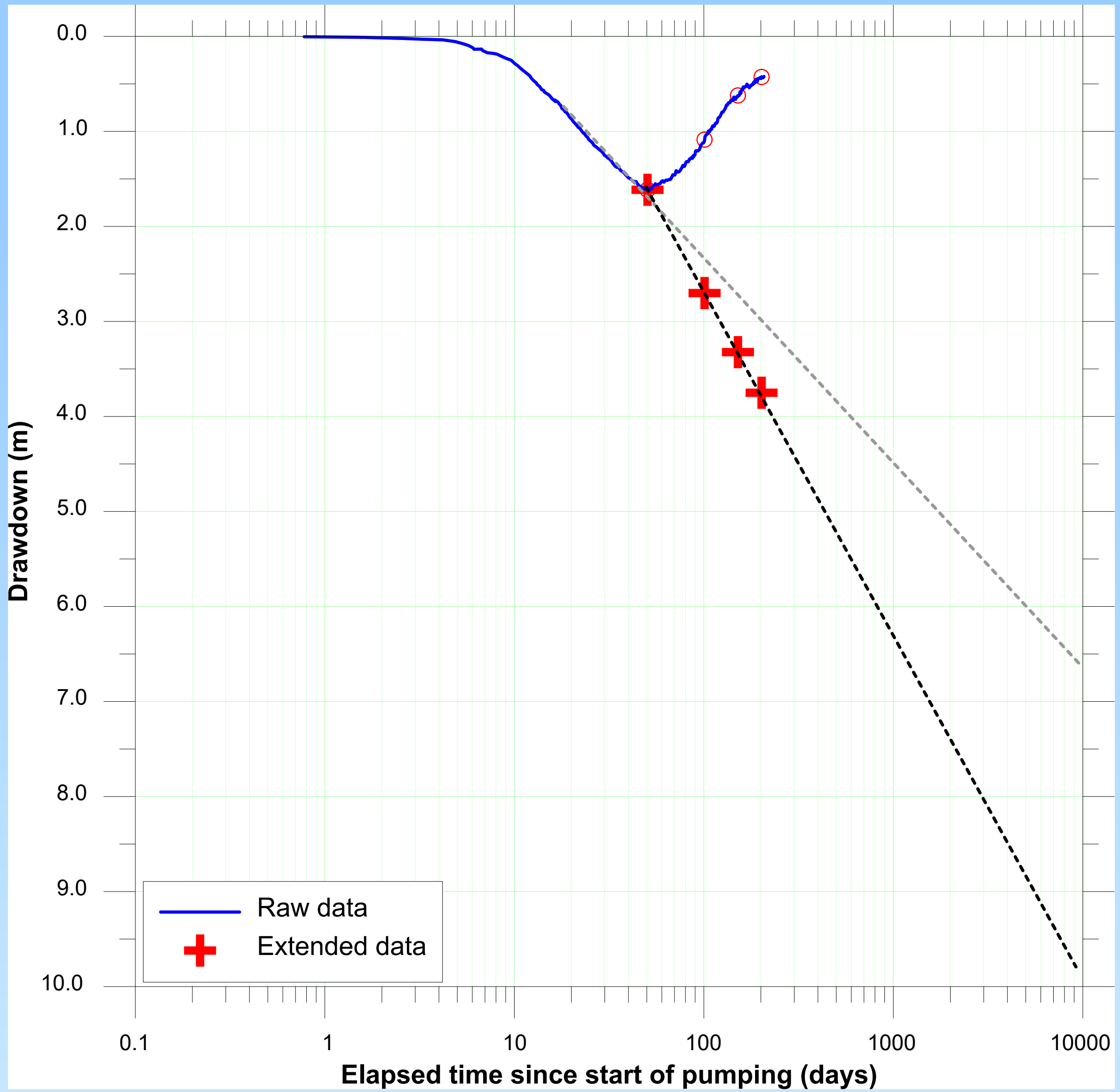


Q: What drawdowns would have been observed if pumping had continued at a constant rate?


Drawdown (m)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0
14
28
42
56
70
84
98
112
126
140
154
168
182
196
210
Elapsed time since start of pumping (days)
1.1 m
1.65 m
Raw data
Extended data
1.1 m + 1.65 m = 2.75 m
0.60 m + 2.75 m = 3.35 m
0.40 m + 3.35 m = 3.75 m



Drawdown (m)
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
0.1
1
10
100
1000
10000
Elapsed time since start of pumping (days)


0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
Drawdown (m)
0.1
1
10
100
1000
10000
Elapsed time since start of pumping (days)
Raw data
Extended data

Take-home points

1. Recovery data may be some of the best data available.
2. Recovery data are straightforward to interpret using superposition.
3. Recovery data may offer insights into aquifer behaviour not available from drawdown data.
4. The van der Kamp method to use of recovery data to extend the effective duration of pumping is simple and useful.